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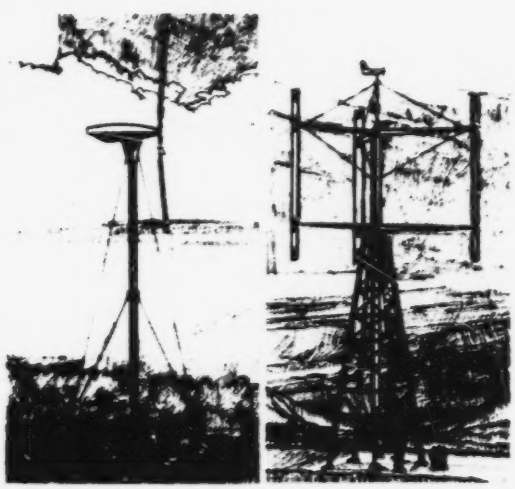
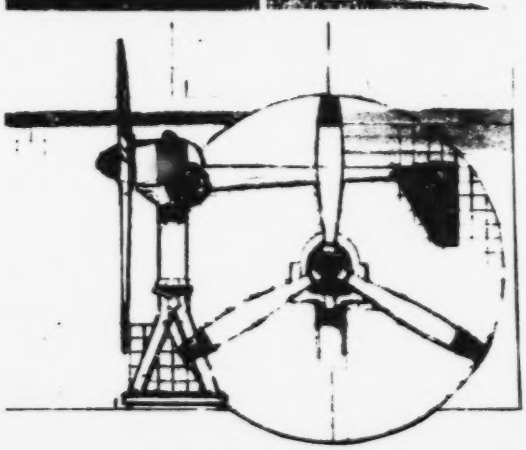
TOWER DYNAMICS ANALYTICAL MODELS

TOWER DYNAMICS ANALYTICAL MODELS: Comparison and Evaluation





December, 1981

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Prepared by the
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UNITED STATES DEPARTMENT OF ENERGY
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Comparison and Evaluation

A.D. Wright

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Prepared by

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ABSTRACT

Several simple tower dynamic analysis methods and computer codes are described and evaluated in this report, by comparison with tower test results.

The theory of these methods is first described. The Rayleigh and Rayleigh-Ritz methods, finite element method, and matrix iteration method are presented. Then, four different towers are described, to show the correct data input for the methods.

The test methods and test results for each tower are given and results for two freestanding and two guyed towers shown. The effects of changes in various parameters on tower natural frequencies are described including tower top weight, guy wire dimensions, and guy wire tension. The effects of errors in measurements of these parameters are also presented.

A comparison of results from each method to tower test results is given. Recommendations are made as to which methods give adequate results for various towers. Large finite element codes are not described here, but recommendations are given as to when they may be needed. A listing and description of these simple computer codes is given in the Appendices.

ACKNOWLEDGEMENT

The author wishes to thank the Massachusetts Institute of Technology and Oregon State University for their kind permission to reproduce the Rayleigh, MITGUY, ISCOE and VIBES structural dynamics computer programs in the Appendices of this report.

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EXECUTIVE SUMMARY

INTRODUCTION

This report evaluates and intercompares several simple tower dynamic analysis methods by comparing computed results to actual tower test results. There is a need for verified simple analyses for designers and analysts who do not have access to large and expensive finite element computer codes. In many cases these simpler methods give results which are useful for preliminary design purposes. This report presents some of these analyses; their correct use and accuracy for various types of towers. The theory of the analytical methods and computer codes is first described. The use of each method is shown for the analysis of a particular tower at Rocky Flats, a freestanding pole tower. Then, the results from each method are compared to tower test results for a number of towers.

The first method discussed is the Rayleigh method. The Rayleigh quotient is shown for a cantilever beam with variable mass and stiffness distributions. A simple polynomial can be used to approximate the first bending modeshape of a cantilever beam. The simple hand computation method for cantilever beams (freestanding towers) with linear mass and stiffness distributions gives good results for uniform beams but poor results for beams with tapered stiffness distributions. A computer program which uses this method can be used for beams (towers) which have nonlinear mass and stiffness distributions.

The Rayleigh-Ritz method, an extension of the Rayleigh method, gives more accurate results, especially for towers which have tapered stiffness and mass distributions. Both a hand computation method and a computer code have been developed using this technique. The machine is treated simply as a lumped mass at the tower top.

Another technique models the tower as a single finite beam element (Programs ISCOE and VIBES). This method was written for use on a Texas

Instruments handheld calculator (TI-59) and takes into account various rotor-nacelle properties such as machine center of gravity, rotor spin rate, and inertial effects. The method can be used to find first and second bending frequencies and first torsional frequencies of both free-standing and guyed towers.

The last method--matrix iteration--is used by Program MITGUY, a computer code. This program requires only a rough estimate of the tower first and second bending modeshapes for very accurate determination of the first and second bending frequencies.

COMPARISON WITH TEST RESULTS

The physical properties required for accurate determination of data input have been defined for four different towers tested at Rocky Flats, including two freestanding and two guyed towers. The freestanding towers are a COR-TEN pole tower and a Rohn SSV truss tower. The guyed towers are a Rohn 25G guyed tower and a COR-TEN guyed tower.

Freestanding Towers

The Rayleigh method gives poor results for the COR-TEN freestanding tower because of its highly tapered stiffness distribution. The Rayleigh-Ritz procedure is much more accurate because a set of assumed shape functions can be used rather than just a simple polynomial to approximate the tower modeshape. The Rayleigh-Ritz hand computation method increases the accuracy of the first bending frequency, while the Rayleigh-Ritz computer code increases the accuracy of both the first and second bending frequencies.

The TI-59 handheld calculator code gives very good estimates for both the first and second bending frequencies of the COR-TEN freestanding tower, as

does the matrix iteration method (Program MITGUY). Matrix iteration is somewhat easier to use because only a rough estimate of the first and second bending modeshapes is needed to give these accurate results.

The analysis of the Rohn SSV freestanding truss tower is more difficult than the COR-TEN tower because the effects of cross braces are hard to account for. The first three methods give large errors for the first fundamental frequency of this tower (see Table I). All of the methods, except for the large finite element code SAPIV, give poor results for the second mode. For this type of tower the last three methods give good results for the fundamental frequency. For this type of tower, a large finite element code is needed for accurate determination of the higher modal frequencies.

Guyed Towers

The methods presented in this report can only be used to analyze guyed towers if no guy wire-tower coupling occurs, such as for the Rohn 25G tower (see Table I). All of the methods, except the Rayleigh method, give good results for the fundamental bending frequency. The Rayleigh method, as developed in this report, can only be used for freestanding towers. The methods also give fair to good results for the second frequency.

The guy wires must be tensioned sufficiently so that guy wire tower interaction will not occur. If this condition is not satisfied, a much more complicated coupled guy wire tower analysis will be needed. Such an analysis was not available for this report.

Effects of Errors or Changes in Parameter Estimates

Changes or errors in estimates of several parameters can have important effects on computed tower natural frequencies. For a Rohn 45 GSR double-guyed tower, increased tower top mass decreases the first and second bending frequencies as seen in the figure. A sensitivity analysis has

revealed that errors in estimating such parameters can have significant effects. In the case of tower top mass, it is suggested that the SWECS designer determine a range of possible tower frequencies by estimating the largest and smallest tower top weights possible.

Another effect of parameter change is noted on guyed towers, where the tower first bending frequency increases as the guy attachment level is raised and the guy wire stiffness is increased. It must be emphasized that these results are only valid when the guy wires are tensioned to prevent guy wire-tower coupling.

CONCLUSIONS

All the methods assessed give especially good results for pole towers such as the COR-TEN, as shown in Table I. The methods also give good results for guyed towers when no guy wire-tower coupling occurs, as in the case of the Rohn 25G. However, for a truss-type tower such as the Rohn SSV, the methods give less accurate results. None of the methods give good results for the second bending frequency of this tower, and only the last three methods in Table I give adequate results for the first bending frequency.

This report was written to introduce members of the wind energy industry to simple tower dynamic analysis methods and their correct use, as well as the accuracy of these methods. The results show that huge and expensive finite element codes are not always needed for accurate determination of tower vibration frequencies and modeshapes.

RECOMMENDATIONS

This study indicates that better simplified methods need to be developed to analyze truss towers, such as the Rohn SSV. This is particularly true for accurate determination of the second bending frequencies of such towers. In addition, better methods need to be developed to analyze guyed towers,

TABLE I: SUMMARY OF ERRORS FOR THE FIRST AND SECOND BENDING FREQUENCY RESULTS FOR EACH METHOD COMPARED TO TEST RESULTS

Methods	COR-TEN Freestanding Tower		Rohn SSV Freestanding Tower		Rohn 25 G Guyed Tower	
	p_1:	p_2:	p_1:	p_2:	p_1:	p_2:
Simple Rayleigh Hand Method	16%	—	19%	—	—	—
Rayleigh Computer Code	19%	—	21%	—	—	—
Simple Rayleigh-Ritz Hand Method	6%	30%	12%	76%	8%	16%
Rayleigh-Ritz Computer Code	3%	1%	6%	22%	0%	13%
Programs ISCOE and VIBES	3%	4%	9%	28%	0%	10%
Program MITGUY	3%	2%	6%	17%	0%	10%

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especially when guy wire-tower interaction occurs. A coupled guy wire-tower dynamic analysis technique would facilitate development of such a method.

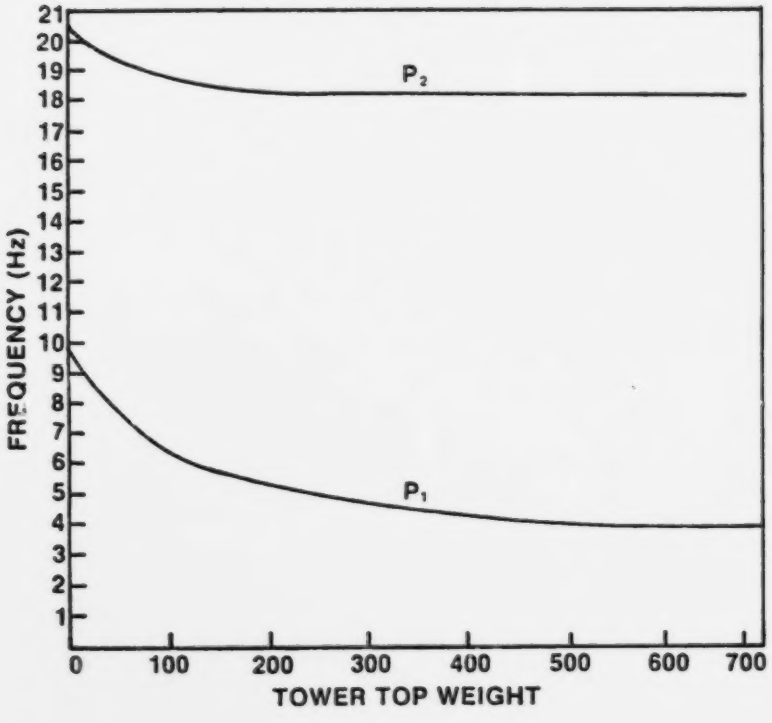

Figure 1: Tower Top Weight Effects for a Rohn 45 GSR Double-Guyed Tower

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Nomenclature List

a - Guy Wire Attachment Level

A' - Guy Cable Cross Sectional Area

E' - Guy Cable Elastic Modulus

$EI(x)$ - Tower Bending Stiffness at x

EI_0 - Tower Base Stiffness

$f(\xi)$ - $EI(L\xi)/EI_0$

$GJ(x)$ - Tower Torsional Stiffness at x

$h(\xi)$ - $\rho A(L\xi)/\rho A_0$

I_m - Tower Mass Moment of Inertia

I_r - Rotor Mass Moment of Inertia

K_c - Guy Wire Stiffness Coefficient

$\bar{K}_c$ - Dimensionless Form of K_c:

$$\bar{K}_c = \frac{K_c L^3}{EI_0}$$

L - Tower Height

ℓ - Guy Cable Length

M - Lumped Mass at Tower Top

M_n - Mass of Nacelle

M_r - Mass of Rotor

p - Bending Frequency

S - Strain Energy

T - Kinetic Energy

t - Time

$V(x,t)$ - Tower Deflection

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W - Dimensionless Frequency:

$$W = \sqrt{\frac{\rho A_0 L^4}{EI_0}}\, p$$

A_i - Coefficients In Approximate Modeshape

m_{ij} - Mass Coefficients

k_{ij} - Stiffness Coefficients

ψ_i - Functions for Approximating Modeshapes

β - Rate of Decrease of Linear Stiffness Distribution

γ - Rate of Decrease of Linear Mass Distribution

ξ - x/L

ξ_a - a/L

θ - Angle Guy Cable makes with Ground

μ - Top Mass Ratio: $M/\rho A_0 L$

$\rho A(x)$ - Mass Distribution of Tower at Section x

ρA_0 - Mass Distribution of Tower at Tower Base

$\phi(x)$ - Tower Modeshape

$\bar{\phi}(x)$ - Particular Approximate Modeshape Used in Rayleigh Method

Ω - Rotor Speed

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1.0 INTRODUCTION

The accurate prediction of tower vibration frequencies and modeshapes is important in avoiding unwanted vibration problems in wind turbines. In many cases, the principal limiting design parameter for a tower proves to be the avoidance of resonance conditions.

Tower resonance problems can be produced in several different ways. If the wind turbine rotor has some small static unbalance present, then an oscillating force with frequency equal to the rotor speed may act on the tower. If the tower natural frequency is equal to the rotor speed, large tower deflections may occur. Also, tower shadow and windshear effects may cause tower resonance problems. The fluctuating air loads on the blades, due to tower shadow or windshear, cause fluctuating blade flapping loads. These cyclical loads can be transmitted to the tower and cause tower resonance if the tower natural frequencies are certain multiples of the rotor speed. In general, it is desirable to avoid having a tower natural frequency equal to any integer multiple of the rotor speed, in the operating range of the machine.

At present there are a variety of structural dynamic analysis techniques for determination of tower natural frequencies. In this report, some of the simpler analyses and computer codes will be examined in order to determine those which can be used by the Small Wind Energy Conversion Systems designer.

There is a need for simplified analyses which have been verified by comparing the results from these methods to actual tower test results. These simplified methods are useful to designers and analysts who do not have access to large and expensive finite element computer codes. They are also easier to use and require less data input and are very useful to designers who need an answer as quickly as possible at low cost. In many cases these simpler methods give results which are useful enough for preliminary design purposes.

A tower dynamics Supporting Research and Technology Project has been conducted at the Rocky Flats Wind Systems Center. At the AWEA meeting in Hyannis, Massachusetts, in 1978, a great need was expressed for simplified and verified tower dynamics analysis techniques. The objective of this project has been to determine those analyses which are simple to use and give adequate results compared to test results. This report presents some of these analyses; their correct use and accuracy for various types of towers.

The first method that will be presented is the Rayleigh method, from which a simple hand computation method is developed. This method gives an estimate of the fundamental bending frequency of a freestanding tower. Also a computer code which uses a more complex version of this method will be presented.

The next method presented is the Rayleigh-Ritz method, an extension of the Rayleigh method. Both a simple hand method and a computer code will be presented, which can both be used for freestanding or guyed towers.

A hand calculator code which can be used to find bending frequencies and the first torsional frequency of guyed or freestanding towers is presented. This analysis uses a method similar to the Rayleigh-Ritz or Finite Element method. In addition, this code takes into account the effects of rotor-nacelle c.g. location, rotor rotation rate, and various other properties of the rotor and nacelle. The previous methods treat the rotor and nacelle simply as a lumped mass at the tower top.

The last method presented is the Matrix Iteration (Stodola) method which uses an iteration technique to obtain accurate values of frequencies and modeshapes. A computer program will be presented which uses this technique.

A large finite element program (such as SAPIV) is not described in this report, since the objective is to describe simple techniques. However, the results from the simple methods is compared to results from a large finite element code for a particular type of tower. These results show when the simple methods give inadequate results compared to test results, and when a large finite element code is needed.

Work done by researchers at Oregon State University (Ref. 9) has led to the development of the hand calculator codes, presented in this report. The development of these codes has been part of a larger project designed to develop analysis techniques for dynamic response of horizontal-axis wind turbines to atmospheric turbulence.

Researchers at Massachusetts Institute of Technology (Ref. 7) have developed two of the codes in this report. Program Rayleigh uses the Rayleigh method for tower analysis and Program MITGUY uses the Matrix Iteration method.

Johnson (Ref. 5) has developed equations for the transverse, axial, and torsional vibrations of pole towers. Also, equations have been developed for vibrations of guy cables. Calculated natural frequencies for a guyed pole tower, supporting a machine, are also presented, showing the effects of guy pretension level on tower natural frequencies. Johnson shows how to treat the problem of guy wire tower interaction.

In the next section of this report, the Rayleigh method will be developed specifically for freestanding towers. A tower will be modeled as a beam. Then, a simple hand analysis for towers will be shown.

2.0 THEORY OF THE ANALYTICAL METHODS AND COMPUTER CODES

2.1 THE RAYLEIGH QUOTIENT

The Rayleigh quotient forms the basis for two approximate methods: the Rayleigh method and the Rayleigh-Ritz method, which are described in this report. The tower is modeled as a beam with distributed mass and stiffness $\rho A(x)$ and $EI(x)$, respectively. The tower has a length L and also has a concentrated mass, M, attached at the top. M represents the tower top machine mass. Uncoupled bending deflections can be modeled by this method, by assuming that the deflections are small (for very flexible towers this assumption may not be valid).

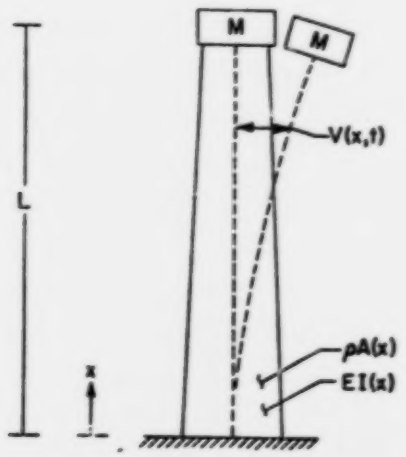

Figure 2.1: Cantilever Beam Tower Model

Before the Rayleigh quotient can be obtained, the tower kinetic and potential energies must first be expressed. The tower and top mass kinetic energy can be shown to be (Ref. 6):

$$T = \frac{1}{2}\int_0^L \rho A(x)\left(\frac{\partial V(x,t)}{\partial t}\right)^2 dx + \frac{1}{2} M\left(\frac{\partial V(L,t)}{\partial t}\right)^2 \qquad 2.1)$$

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where:

$\frac{\partial V(L,t)}{\partial t}$ represents the velocity of the tower top, and

$\frac{\partial V(x,t)}{\partial t}$ represents the velocity of a particle attached to the tower axis.

The tower potential energy (or strain energy) can be shown to be (Ref. 6):

$$S = \frac{1}{2} \int_0^L EI(x) \left(\frac{\partial^2 V(x,t)}{\partial x^2} \right)^2 dx. \qquad 2.2)$$

For harmonic motion of the tower, the deflection is assumed in the form:

$$V(x,t) = \bar{V}\, \phi(x)\, \sin pt, \qquad 2.3)$$

where p is the frequency of harmonic motion, $\phi(x)$ is the corresponding vibration modeshape, and $\bar{V}$ is the amplitude of the motion.

By equating the tower potential energy at maximum displacement to the kinetic energy at minimum displacement (and assuming no damping in the system), the Rayleigh quotient can be expressed as:

$$p^2(\phi) = \frac{\int_0^L EI(x)\, [\phi''(x)]^2\, dx}{\int_0^L \rho A(x)\, [\phi(x)]^2\, dx + M[\phi(L)]^2} \qquad 2.4)$$

When ϕ is an exact vibration mode shape of the tower, p has the value of the corresponding exact bending frequency; also, p^2 is stationary with respect to variations in ϕ at each natural frequency (Ref. 6). This expression can be expected to give a good approximation to the frequency if a good approximation to the exact mode shape ϕ is used to evaluate 2.4. A simple analysis method can be used to find the first bending frequency by

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using a polynomial to approximate the first bending mode shape of a free-standing tower. This method is especially useful for towers with uniform or linearly varying mass and stiffness distributions.

2.2 THE RAYLEIGH METHOD

For a freestanding tower a simple polynomial $\bar{\phi}$ can be used to evaluate the Rayleigh quotient (Equation 2.4):

$$\bar{\phi}(x) = \left(\frac{x}{L}\right)^4 - 4\left(\frac{x}{L}\right)^3 + 6\left(\frac{x}{L}\right)^2 \qquad 2.5)$$

The assumed mode shape should satisfy at least the tower geometric boundary conditions and should also closely approximate the first bending mode-shape (see Figure 2.3). In this case $\bar{\phi}$ was chosen to satisfy the base boundary conditions of a cantilever beam, i.e., that of zero displacement and zero slope at the base ($\bar{\phi}(o) = 0$, $\bar{\phi}'(o) = 0$). For a pinned beam a different approximating function would have to be chosen (to satisfy zero displacement and zero curvature at the base, $\bar{\phi}(o)=0$, $\bar{\phi}''(o)=0$).

This method is particularly useful for towers which have linearly varying mass and stiffness distributions of the form:

$$EI(x) = EI_o\,(1 - \beta\, x/L)$$
$$\rho A(x) = \rho A_o\,(1 - \gamma\, x/L), \qquad 2.6)$$

where β is the rate of decrease of linear stiffness and γ is the rate of decrease of linear mass distribution as shown in Figure 2.2.

The approximate modeshape 2.5) and the expressions for stiffness and mass distributions 2.6) can be used to evaluate 2.4) with the result:

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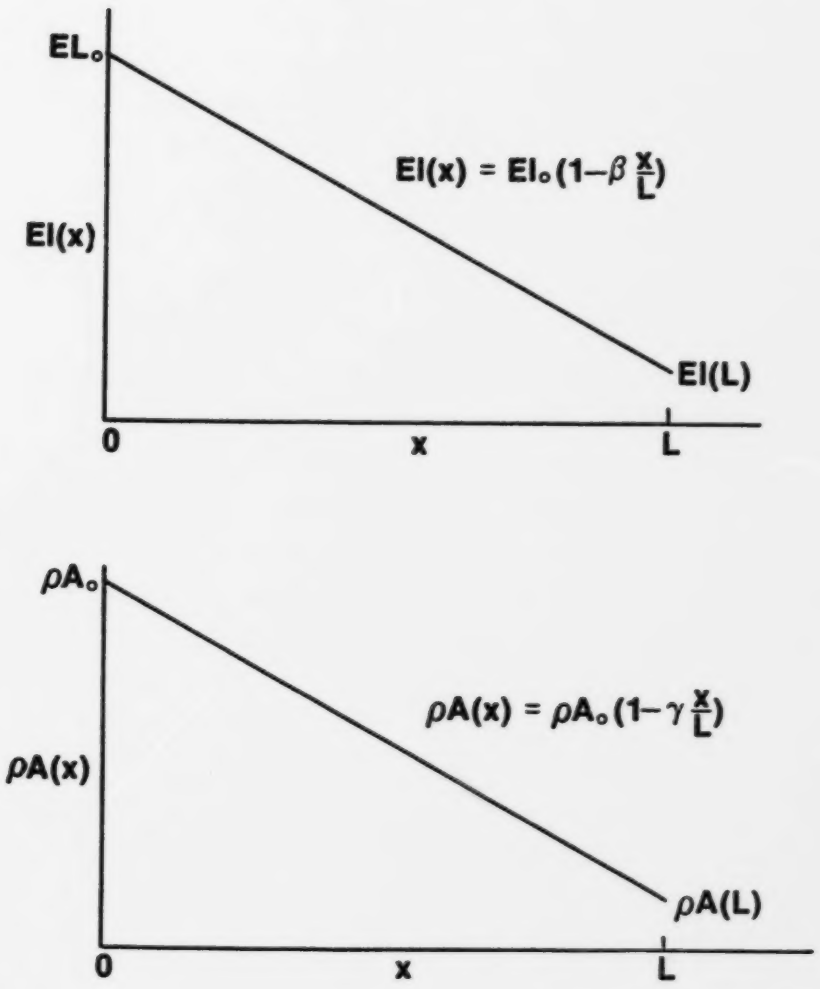

Figure 2.2: Linear Mass and Stiffness Distributions

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$$p^2 = \sqrt{\frac{EI_o}{\rho A_o L^4}} \quad \frac{28.800 - 4.800\beta}{(2.311 - 1.854\gamma) + 9.00\mu} \qquad 2.7)$$

where μ is the top mass ratio $M/\rho A_o L$.

It should be noted that the values of β, γ, and μ can be varied in 2.7), giving frequency estimates for beams with different linear stiffness and mass distributions, and tower top masses. Table I shows the dimensionless frequency ratio:

$$W = \sqrt{\frac{\rho A_o L^4}{EI_o}}\ p$$

obtained from 2.7), compared to exact results calculated, using a power series method (Ref. 10), for a simple cantilever beam.

From Table I it can be seen that the results from this simple method are higher than the exact results. The beam, which actually has infinite degrees of freedom, is being modeled with one degree of freedom. Constraints are added to the system which causes the resulting frequency estimate to be high. As will be seen in later sections, the methods using more degrees of freedom give more accurate results.

Also from Table I it can be seen that the accuracy of results from 2.7) is poor for beams in which the stiffness distributions taper to nearly a point at the tower top (β = .9). As can be seen in Figure 2.3, the assumed shape $\bar{\phi}$ approximates the exact mode shape (calculated from Ref. 10) inaccurately, for β = 0.9.

In Figure 2.3 three different modeshapes are shown. The approximating polynomial $\bar{\phi}$ is shown as well as modeshapes calculated from Ref. 10, for a beam with uniform mass and stiffness distribution (β = 0.0, γ = 0.0), and a beam with uniform mass but a high degree of stiffness distribution taper (β = 0.9, γ = 0.0). Also in this figure, the effects of the parameter β are shown (top mass ratio).

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TABLE I: COMPARISON OF RESULTS OBTAINED FROM THE SIMPLE RAYLEIGH HAND METHOD (W_s) TO THE RESULTS FROM REFERENCE 10 (W_e), FOR A CANTILEVER BEAM, FOR VARIOUS VALUES OF β, γ, AND μ.

$\beta = 0.$, $\gamma = 0.$ Uniform Stiffness and Mass

μ	W_e	W_s	% Error
1.0	1.56	1.60	2.6
2.0	1.16	1.19	2.6

$\beta = 0.$, $\gamma = .9$ Uniform Stiffness and Tapered Mass

μ	W_e	W_s	% Error
1.0	1.68	1.73	3.0
2.0	1.21	1.24	2.5

$\beta = .9$, $\gamma = 0.$ Tapered Stiffness and Uniform Mass

μ	W_e	W_s	% Error
1.0	1.33	1.47	10.5
2.0	0.99	1.10	11.1

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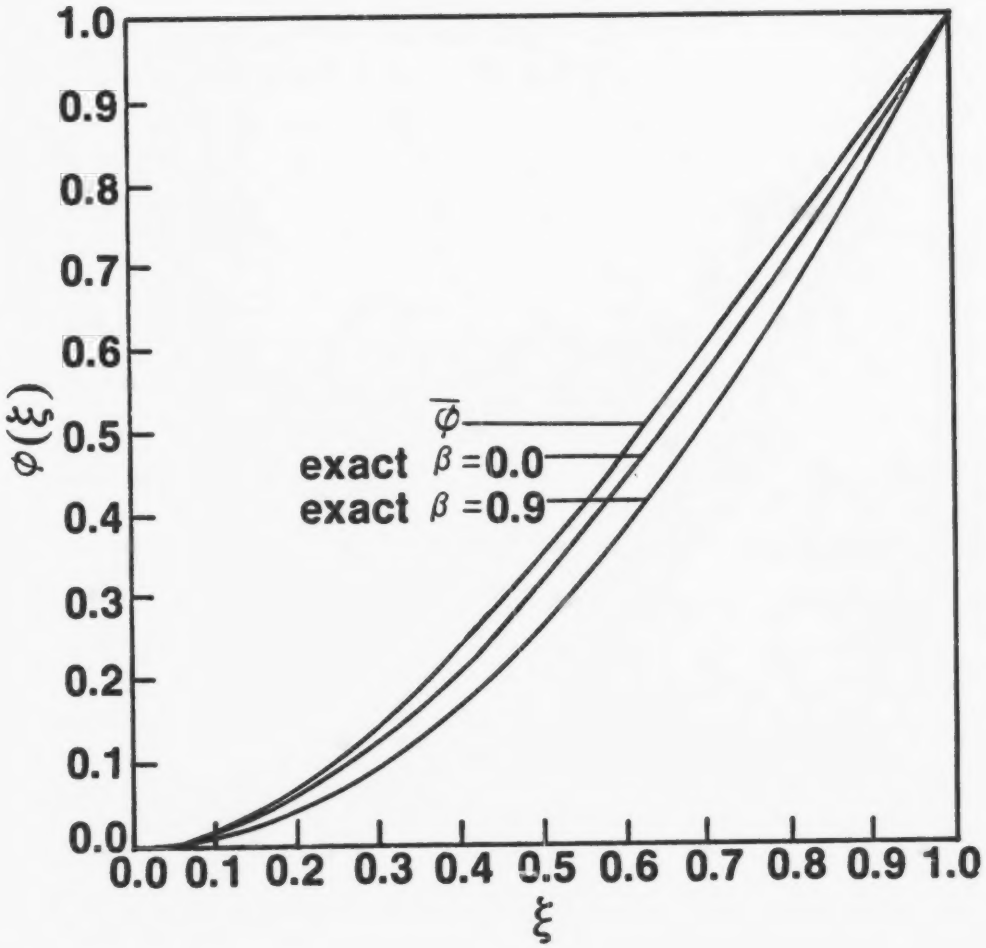

Figure 2.3: Comparison of Assumed Modeshape $\overline{\phi}$ to the Exact Modeshape for $\beta = 0.9$, $\gamma = 0.0$, and $\mu = 1.0$; and $\beta = 0.0$, $\gamma = 0.0$, and $\mu = 1.0$.

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Limitations of the Simple Rayleigh Hand Method

Equation 2.7 can be used to estimate the first bending frequency of freestanding towers, in which the mass and stiffness distributions vary linearly. The results for towers with highly tapered stiffness distributions ($\beta \simeq 0.9$) are inaccurate, because the modeshape approximation $\bar{\phi}$ is poor. This basic method could still be used if the analyst could find a better modeshape approximation. A better idea would be to use a method utilizing more degrees of freedom, as will be seen in the section on the Rayleigh-Ritz method.

For towers having nonlinear mass and stiffness distributions, the functions $\rho A(x)$ and $EI(x)$ could be estimated by polynomials. For towers with erratic distributions this method could become very laborious. Fortunately a computer program has been developed to handle this problem.

A Rayleigh Computer Code

A program developed at MIT, Program Rayleigh (Ref. 7), uses the Rayleigh method to evaluate bending frequencies for freestanding towers. This program is especially useful for towers with nonlinear mass and stiffness distributions.

The tower can be divided into N segments and constant values of mass and stiffness input along each segment. In addition, the user can input any desired modeshape. Numerical integration techniques are then used to evaluate the integrals in the Rayleigh quotient. The use of this program and the input data will be illustrated in Section 5.0 and Appendix A.

Both the simple Rayleigh hand method and the Rayleigh computer code have the disadvantage that the user must input an assumed modeshape. The accuracy of the corresponding frequency results depends heavily on this one shape.

In a method to be described next, a linear combination of assumed shape functions is used to form the function $\phi(x)$ for use in the Rayleigh quotient. This method utilizes more degrees of freedom, and thus gives more accurate results than the one degree of freedom Rayleigh method. This method is particularly useful for guyed towers, because the modeshapes depend, in part, on the guy wire attachment level. The modeshapes are hard to approximate with a simple polynomial such as in the Rayleigh method.

2.3 THE RAYLEIGH-RITZ PROCEDURE

The Rayleigh-Ritz procedure involves using a set of assumed shape functions $\psi_i(x)$ and combining them to form the modeshape approximation:

$$\phi(x) = A_1\psi_1(x) + A_2\psi_2(x) + \ldots + A_N\psi_N(x) \qquad 2.8)$$

The A_i's are parameters to be determined and the ψ_i's are a set of linearly independent (not necessarily orthogonal) functions, each satisfying at least the tower geometric boundary conditions. When N such functions are used, the tower is modeled with N degrees of freedom.

In order to model a guyed tower, the Rayleigh quotient must be modified to take into account the guy wire effects. It can be shown that the guy wires add an extra term to the potential energy of the system (Ref. 9). This extra term appears in the numerator of the Rayleigh quotient:

$$p^2(\phi) = \frac{\int_0^L EI(x)\,[\phi''(x)]^2\,dx + K_c[\phi(a)]^2}{\int_0^L \rho A(x)\,[\phi(x)]^2\,dx + M[\phi(L)]^2} \qquad 2.9)$$

K_c is the guy wire stiffness coefficient and a is the guy wire attachment height, shown in Figure 2.4. It can be shown that for three guy wires spaced 120° apart,

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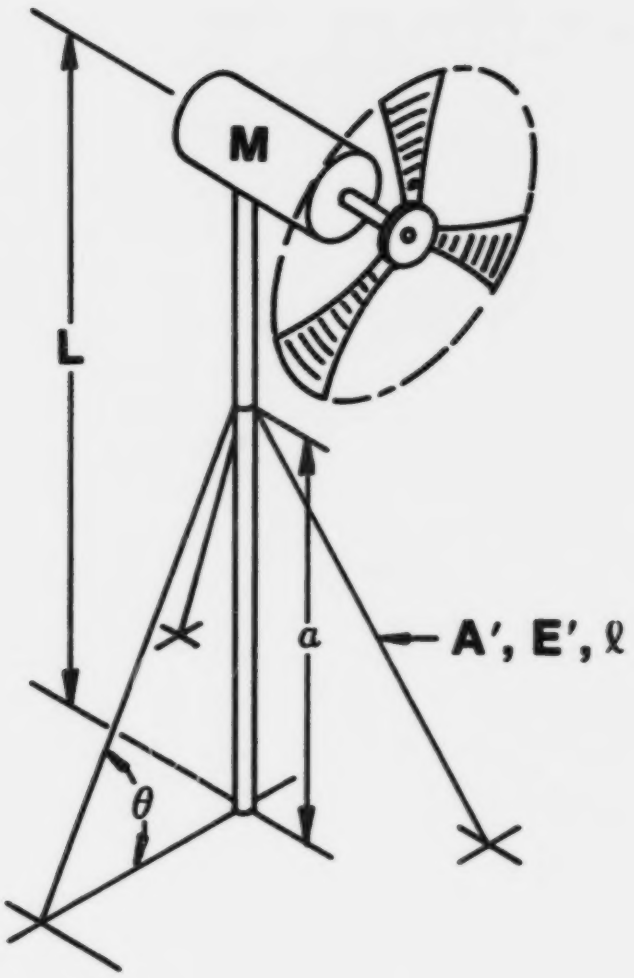

Figure 2.4: Guyed Tower

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$$K_c = \frac{3}{2}\left(\frac{A'E'}{\ell}\right)\cos^2\theta. \qquad 2.10)$$

For four guy wires spaced 90° apart, the factor 3/2 in Equation 2.10) is replaced by 2 (Ref. 9). The guy wires are assumed to be massless. The effects of this assumption are described in Reference 1.

The effects of the guy wires can be modeled in this way only when the guy wires have been tensioned sufficiently so that there is no coupling between the guy wires and tower (Ref. 1). More will be explained about guy wire effects in Section 6.0. For these simple analysis methods, the guy wire effects will be modeled in this way.

The Rayleigh-Ritz method involves the substitution of the parameter laden modeshape approximation 2.8) into Rayleigh's quotient 2.9), with the result:

$$p^2(\phi) = \frac{EI_o}{\rho A_o L^4} \quad \frac{\sum_j \sum_i A_i A_j k_{ij}}{\sum_j \sum_i A_i A_j m_{ij}} \qquad 2.11)$$

Where:

$$k_{ij} = \int_0^1 f(\xi)\,\psi_i''(\xi)\psi_j''(\xi)d\xi + \bar{K}_c\psi_i(\xi_a)\psi_j(\xi_a)$$

$$m_{ij} = \int_0^1 h(\xi)\,\psi_i(\xi)\psi_j(\xi)d\xi + \mu\psi_i(1)\psi_j(1) \qquad 2.12)$$

$$i, j = 1, 2, \ldots, N.$$

and:

$$f(\xi) = EI(\xi)/EI_o$$

$$h(\xi) = \rho A(\xi)/\rho A_o$$

$$\bar{K}_c = \frac{K_c L^3}{EI_o},\ \xi = x/L$$

$$\xi_a = a/L \text{ and } \mu = M/\rho A_o L.$$

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The terms k_{ij} and m_{ij} are presented in dimensionless form. The analyst should determine the tower stiffness and mass distributions in the form:

$$EI(\xi) = EI_0 \, f(\xi)$$
$$\rho A(\xi) = \rho A_0 \, h(\xi)$$

where EI_0 and ρA_0 are the stiffness and mass distributions of the tower at the base. $\bar{K}_c$ is the dimensionless guy stiffness coefficient, formed by multiplying K_c by the cube of the tower height and dividing by the base stiffness. ξ_a is the dimensionless attachment level, formed by dividing a by the tower height L.

It is known that the Rayleigh quotient is stationary with respect to variations in $\phi(x)$, when $\phi(x)$ is an exact modeshape for the tower. This can be shown to require that:

$$\text{Det} \, [k_{ij} - W^2 m_{ij}] = 0 \qquad \text{(Ref. 3)}$$

Where:

$$W = \sqrt{\frac{\rho A_0 L^4}{EI_0}} \, p$$

is the dimensionless frequency ratio. The analyst then multiplies W by

$$\sqrt{\frac{EI_0}{\rho A_0 L^4}}$$

to get the tower frequency in radians per second.

The Rayleigh-Ritz problem is thus reduced to finding the values of W^2 which make this N x N determinant equal to zero. Once the N values of W^2 have been found, the modeshape, corresponding to a particular vibration frequency can be found by using the N x N matrix to find the N coefficients $A_1, A_2, \ldots, A_N$. Once these coefficients have been determined, the modeshape approximation is given by:

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$$\phi(x) = A_1 \psi_1(x) + A_2 \psi_2(x) + \ldots + A_N \psi_N(x),$$

where the $\psi_i(x)$ functions are already known. More details will be given in Section 5.0, where an example tower is analyzed for frequencies and modeshapes. A simple hand analysis can be formed by using two degrees of freedom, i.e., two approximating ψ_i functions.

A Simple Rayleigh-Ritz Hand Analysis

A simple two-mode analysis for towers can be developed by using the approximating function for the Rayleigh method:

$$\psi_1(\xi) = \xi^4 - 4\xi^3 + 6\xi^2 \qquad 2.13)$$

as the first degree of freedom function, as well as the function:

$$\psi_2(\xi) = 3\xi^5 - 10\xi^4 + 10\xi^3$$

for the second degree of freedom. As will be seen later, the use of the second degree of freedom function ψ_2 will greatly improve the first bending frequency results compared to the results from the simple Rayleigh hand method in Table I. These functions can only be used for freestanding cantilever towers. For hinged base towers (supported by guy wires) functions need to be chosen which satisfy the pinned beam boundary conditions, ($\psi(o) = 0$, $\psi''(o) = 0$).

For towers with linear mass and stiffness distributions as given in Equation 2.6) the k_{ij} and m_{ij} integrals are:

$$\begin{aligned} k_{11} &= 28.800 - 4.800\beta + \bar{K}_c\,[\psi_1(\xi_a)]^2 \\ k_{12} &= 24.000 - 6.857\beta + \bar{K}_c\,\psi_1(\xi_a)\,\psi_2(\xi_a) \\ k_{22} &= 34.286 - 12.857\beta + \bar{K}_c\,[\psi_2(\xi_a)]^2 \end{aligned} \qquad 2.14)$$

$$\begin{aligned} m_{11} &= 2.311 - 1.854\gamma + 9.00\mu \\ m_{12} &= 2.070 - 1.700\gamma + 9.00\mu \\ m_{22} &= 1.882 - 1.573\gamma + 9.00\mu. \end{aligned}$$

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When det $[k_{ij} - W^2 m_{ij}]$ is evaluated, the resulting polynomial:

$$g(W^2)^2 - qW^2 + 1 = 0 \qquad 2.15)$$

must be solved where:

$$g = m_{11}m_{22} - m_{12}^2$$
$$q = k_{11}m_{22} + m_{11}k_{22} - 2k_{12}m_{12}$$
$$1 = k_{11}k_{22} - k_{12}^2$$

This simple hand analysis consists of evaluating the k_{ij} and m_{ij} terms, for a tower with a given stiffness and mass distribution, tower top mass, guy wire size, and attachment height. For freestanding towers, the K_c terms are set to zero. Once the k_{ij} and m_{ij} terms have been calculated, the polynomial 2.15) can be solved for W. The evaluation of the k_{ij} and m_{ij} terms by hand becomes difficult, if the EI and ρA functions are nonlinear. For this reason a computer code has been developed at Rocky Flats.

The Rayleigh-Ritz Computer Code

The Rayleigh-Ritz method can be used to determine any number of modal frequencies; however, the computational complexity increases greatly after the first couple of modes. Also, for towers with highly nonlinear mass or stiffness distributions, evaluation of the stiffness and mass terms in 2.12) becomes too difficult. For these reasons a four degree of freedom method has been developed using the two functions ψ_1 and ψ_2 of the last section and also the two additional functions:

$$\psi_3(\xi) = 2\xi^6 - 6\xi^5 + 5\xi^4$$
$$\psi_4(\xi) = \xi^{10} - 2.5\xi^9 + 1.607\xi^8.$$

The program can be used for towers with nonlinear or linear mass and stiffness distributions by dividing the tower into N sections and inputting

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constant values of mass and stiffness along each section. Numerical integration techniques are then used to evaluate the integrals in the Rayleigh quotient.

Table II shows results for the first two bending frequencies of a free-standing tower, using the simple Rayleigh-Ritz hand analysis (W_{12}, W_{22}) and 4 degree of freedom computer code (W_{14}, W_{24}).

TABLE II: COMPARISON OF RESULTS FROM: 1) THE SIMPLE RAYLEIGH-RITZ HAND ANALYSIS, (W_{i2}); AND 2) THE RAYLEIGH-RITZ COMPUTER CODE, (W_{i4}); TO THE EXCACT RESULTS OF REF. 10, (W_{ie}), FOR A CANTILEVER BEAM. VARIOUS VALUES OF β, γ, AND μ ARE SHOWN.

β = 0., γ = 0. Uniform Stiffness and Uniform Mass

μ	W_{1e}	W_{12}	W_{14}	W_{2e}	W_{22}	W_{24}
1.0	1.56	1.56	1.56	16.25	17.61	16.26
2.0	1.16	1.16	1.16	15.86	17.29	15.88

β = 0., γ = .9 Uniform Stiffness and Tapered Mass

μ	W_{1e}	W_{12}	W_{14}	W_{2e}	W_{22}	W_{24}
1.0	1.68	1.70	1.70	22.37	23.56	22.49
2.0	1.21	1.21	1.21	22.17	23.00	22.30

β = .9, γ = 0. Tapered Stiffness and Uniform Mass

μ	W_{1e}	W_{12}	W_{14}	W_{2e}	W_{22}	W_{24}
1.0	1.33	1.37	1.34	11.81	15.36	12.75
2.0	0.98	1.02	0.99	11.53	15.06	12.45

As can be seen from this table, the simple Rayleigh-Ritz hand analysis gives poor results for the second bending frequencies of towers with tapered stiffness distributions (β = 0.9). The more complex four-mode Rayleigh-Ritz computer code gives much better results. This code can be used for towers having more complex mass and stiffness distributions.

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In the methods just discussed, the nacelle and rotor were modeled as a single lumped mass at the tower top. As will be seen in the next section, the rotor-nacelle moments of inertia, the machine c.g. location, and the rotor spin rate can have an effect on the natural frequencies of a tower-machine system. The programs to be described next take these effects into account. They are designed to be used on a hand-held calculator.

2.4 PROGRAMS ISCOE AND VIBES (A Single Finite Element Method)

These programs have been developed at Oregon State University for use in estimating the natural frequencies of wind turbine systems. They can be used on a programmable TI-59 (Texas Instruments) calculator.

Figure 2.5 shows a diagram of a model of the wind turbine system, which has a symmetrical rotor and a guy-stiffened tower. The rotor and nacelle are assumed to be a single rigid body which, due to the tower flexibility, is free to rotate in pitch and yaw and to translate in a plane parallel to the ground. The yaw angle of the rotor axis is defined by the angle Φ, and the pitch angle by χ. The lateral translation U, is in the direction of the x coordinate, and the V translation is in the Y direction along the rotor axis. The rotor spin velocity is Ω, and is assumed constant.

The tower has a section L_1, in which the stiffness and mass properties can vary. The section L_2 is assumed to have constant properties (uniform mass and stiffness distributions). The base of the tower may be fixed (cantilever base) or it may be simply supported (balljoint base). This model utilizes the four degrees of freedom mentioned previously: U, V, Φ, and χ. The linear equations of unforced motion are:

FOUR DEGREE OF FREEDOM SYSTEM

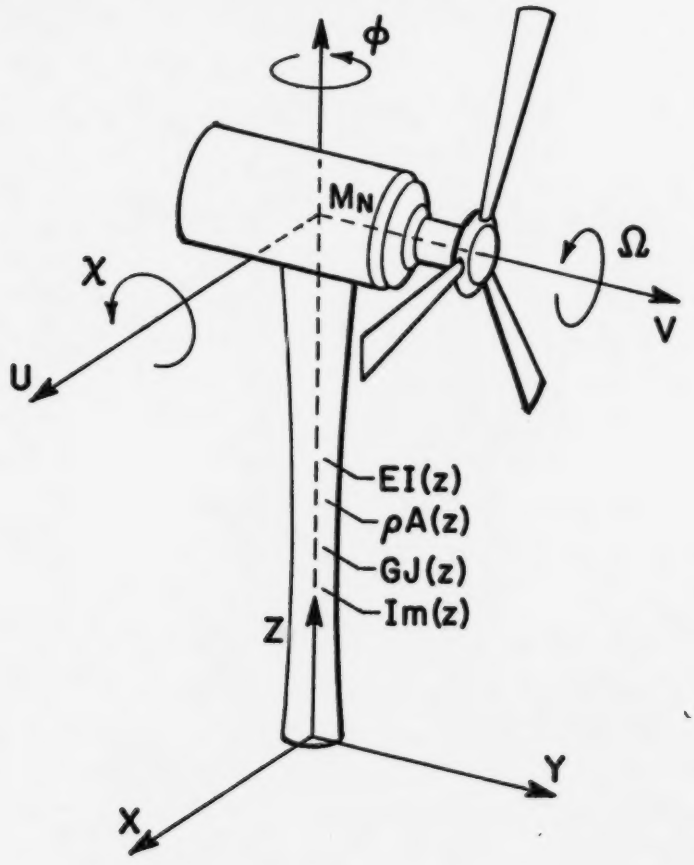

Figure 2.5: Wind Turbine System

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$$\begin{bmatrix} M_{UU} & 0 & M_{U\Phi} & 0 \\ 0 & M_{VV} & 0 & M_{VX} \\ M_{U\Phi} & 0 & M_{\Phi\Phi} & 0 \\ 0 & M_{VX} & 0 & M_{XX} \end{bmatrix} \begin{Bmatrix} U \\ V \\ \Phi \\ X \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega I_r \\ 0 & 0 & -\Omega I_r & 0 \end{bmatrix} \begin{Bmatrix} U \\ V \\ \Phi \\ X \end{Bmatrix}$$

$$+ \begin{bmatrix} K_{UU} & 0 & 0 & 0 \\ 0 & K_{VV} & 0 & K_{VX} \\ 0 & 0 & K_{\Phi\Phi} & 0 \\ 0 & K_{VX} & 0 & K_{XX} \end{bmatrix} \begin{Bmatrix} U \\ V \\ \Phi \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (2.16)$$

The mass coefficients M_{UU}, M_{XX} are composed of the contributions due to the rigid nacelle and rotor, and the contributions due to the flexible tower (modeled as a beam). The stiffness coefficients K_{UU}, ..., K_{XX} are composed of the flexible tower effects. These coefficients are included in Appendix C. The tower model will be described so that a feeling for the methods of calculating the tower mass and stiffness coefficients can be attained.

The structural properties of the tower are evaluated, using a single finite element, shown in Figure 2.6. Loads are applied at the tower top with each degree of freedom.

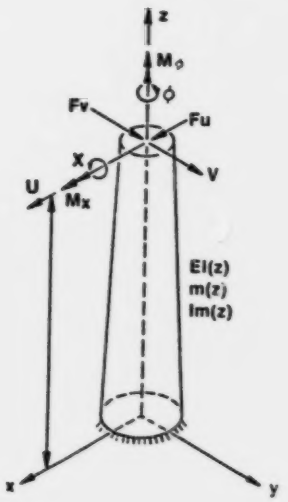

Figure 2.6: A Single Finite Element

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For a complete description of this method, the reader should consult Ref. 9. The displacements of the tower are assumed in the form:

$$V(z,t) = \psi_V(z)\, V(t) + \psi_\chi(z)\, \chi(t) \qquad 2.17)$$
$$\phi(z,t) = \psi_\phi(z)\, \phi(t).$$

The expression of $V(z,t)$ and $\phi(z,t)$ in this form is similar to the Rayleigh and Rayleigh-Ritz methods. The shape functions $\psi_V(z)$, $\psi_\chi(z)$, and $\psi_\phi(z)$ have been selected as:

$$\psi_V(z) = (z/L)^2\,(3 - 2\,z/L)$$
$$\psi_\chi(z) = z(z/L)\,(1 - z/L) \qquad 2.18)$$
$$\psi_\phi(z) = z/L.$$

These functions represent the displacement distributions of a uniform cantilever beam in static equilibrium with forces and couples applied at the end (Ref. 9). They satisfy the geometric boundary conditions of a cantilever beam at the base, that of zero displacement and zero slope at the base. For a guyed tower with balljoint base the functions are:

$$\psi_V(z) = 1/2\,(z/L)\,(3 - (z/L)^2)$$
$$\psi_\chi(z) = z/2\,(1 - (z/L)^2) \qquad 2.19)$$
$$\psi_\phi(z) = (z/L).$$

These functions satisfy the boundary conditions of a beam pinned at the base, i.e., zero deflection and zero moment (Ref. 9). The tower stiffness coefficients can be shown to be:

$$K_{VV} = \int_0^L EI(z)\,[\psi_V''(z)]^2\,dz + K_c\,\psi^2_V(a)$$

$$K_{\chi V} = \int_0^L EI(z)\,\psi_V''(z)\,\psi_\chi''(z)\,dz + K_c\,\psi_V(a)\,\psi_\chi(a) \qquad 2.20)$$

$$K_{\chi\chi} = \int_0^L EI(z)\,[\psi_V''(z)]^2\,dz + K_c\,\psi^2_\chi(a)$$

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$$K_{\phi\phi} = \int_0^L GJ(z)\,[\psi_\phi'(z)]^2\,dz.$$

The mass coefficients for the tower are:

$$m_{VV} = \int_0^L \rho A(z)\,\psi^2_V(z)\,dz$$

$$m_{VX} = \int_0^L \rho A(z)\,\psi_V(z)\,\psi_X(z)\,dz$$

$$m_{XX} = \int_0^L \rho A(z)\,\psi^2_X(z)\,dz$$

$$m_{\phi\phi} = \int_0^L I_m(z)\,\psi^2_\phi(z)\,dz.$$

The reader should note that the calculation of these mass and stiffness coefficients is very similar to calculation of the k_{ij} and m_{ij} coefficients in the Rayleigh-Ritz method shown in Section 2.3.

The coefficients m_{UU} and K_{UU} are obtained by a method called "static condensation," whereby unwanted degrees of freedom are eliminated.

$$K_{UU} = (K_{VV} - K_{VX}{}^2/K_{XX})$$

$$m_{UU} = \int_0^L \rho A(z)\,\psi^2_U(z)\,dz \qquad \text{(Ref. 6)}$$

Where: $$\psi_U(z) = \psi_V(z) - \left(\frac{K_{VX}}{K_{XX}}\right)\psi_X(z)$$

After the tower mass coefficients have been calculated, the complete system inertia coefficients are calculated as follows:

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$$M_{UU} = M_r + M_n + m_{UU}$$

where:

M_r = Mass of the rotor
M_n = Mass of the nacelle

$$M_{U\Phi} = -(M_r + M_n)\, q$$

where:

q = distance from tower centerline to the machine c.g. location

$$M_{VV} = M_r + M_n + m_{VV}$$
$$M_{VX} = m_{VX}$$
$$M_{\Phi\Phi} = I_{zz} + m_{\Phi\Phi}$$

where:

I_{zz} = the mass moment of inertia of the nacelle and rotor about the z axis.

$$M_{XX} = I_{XX} + m_{XX}$$

where:

I_{XX} = mass moment of inertia of the nacelle and rotor about the X axis.

The inertia coefficients are seen to be the sum of the nacelle-rotor inertias and the tower inertias. The stiffness coefficients are those of the tower alone, the nacelle and rotor having been modeled as a rigid body.

The terms ΩI_r are the gyroscopic coupling coefficients, where:

Ω = rotor spin rate

I_r = total effective inertia of all spinning mass about the y axis

$$I_r = \sum_{i=1}^{N} (I_i\, N_i)$$

I_i = individual component mass moments of inertia about its spin axis

N_i = (component rotation rate)/Ω.

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As will be seen in Section 7.0, the accuracy of the method is similar to that of the two-mode Rayleigh-Ritz method. In some cases, such as for towers with tapered stiffness distributions, the second bending frequency results are inaccurate.

The great utility of this program is that it gives good fundamental mode results, and can be used on a hand-held calculator. The program also takes into account rotor and nacelle inertia effects, machine c.g. location, and rotor spin rate.

In the approximate methods just discussed, shape functions have been chosen to approximate the true tower modes of vibration. The frequency results have depended upon how well these shape functions approximated the exact modeshape. In the next section a Matrix Iteration method will be discussed, whereby a previously chosen shape function is modified and improved through iteration techniques. By improving the chosen shape function, the frequency results are forced to converge to the precise values. The previous methods discussed did not involve iteration; the shape functions were used only once, for evaluation of the integrals in the Rayleigh quotient.

2.5 MATRIX ITERATION (Stodola Method)

The Matrix Iteration method is useful for the determination of precise values of frequencies, without the necessity of assuming a precise modeshape approximation. A rough estimate of the corresponding modeshape will result in convergence to the desired frequency by means of iteration.

A computer code which utilizes this method has been developed at MIT (called program MITGUY) for freestanding and guyed towers. The method is based on the equation:

$$\frac{1}{W^2}\{\phi\} = [f][m]\{\phi\} \qquad 2.21)$$

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where [f] represents the matrix of flexibility influence coefficients (Ref. 7) and [m] represents the mass matrix. $\{\phi\}$ represents the exact modeshape at various points along the tower, and W represents the natural frequency of vibration for the particular vibration mode being sought. A rough estimate of $\{\phi\}$ will provide the modeshapes as well as the natural frequencies of the tower, by iteration.

As described in Appendix E, the [f] matrix is obtained by breaking the tower into N equal segments and evaluating the stiffness at the nodes of each segment. The tower is then approximated by a series of constant stiffness steps, where each step value of stiffness for a given segment is found by taking the average of the values of stiffness at the endpoints. The elements of the matrix [f] are obtained from the general expressions in Ref. 7, after the tower stiffness has been estimated. By lumping the mass at the nodes, as shown in Appendix E, a simple diagonal mass matrix is obtained. These operations are performed in the MITGUY computer program, listed in Appendix E.

After determination of the [f] and [m] matrices, one uses Equation 2.21) for the iteration procedure. A trial mode $\{\phi_1\}^0$ is obtained for estimation of the first bending modeshape; it should represent a rough estimate of this shape and can have arbitrary amplitude. The subscript 1 used with this notation identifies the first modeshape; the superscript 0 shows that this is the initial shape assumption.

Equation 2.21) will be satisfied only by those vectors $\{\phi\}$ which represent an exact vibration modeshape of the tower. Since the initial shape assumption $\{\phi_1\}^0$ is not an exact first bending modeshape, and the first bending frequency W_1 is not exactly known, Equation 2.21) cannot be satisfied exactly. The vector $\{\phi_1\}^0$, however, can be substituted into 2.21) with the result:

$$\overline{\{\phi_1\}}^1 = [f]\,[m]\,\{\phi_1\}^0 \qquad 2.22)$$

where $\overline{\{\phi_1\}}^1$ is now a new vector, different from $\{\phi_1\}^0$. A new shape approximation $\{\phi_1\}^1$ is obtained from the vector $\overline{\{\phi_1\}}^1$ by normalizing the values of $\overline{\{\phi_1\}}^1$ so that the deflection at the tower top is one (dividing each value by $K^{(1)}$, the top deflection):

$$K^{(1)} \{\phi_1\}^1 = \overline{\{\phi_1\}}^1$$

The new vector $\{\phi_1\}^1$ can now be used to compute a new shape by introducing it into Equation 2.21):

$$\overline{\{\phi_1\}}^2 = [f] [m] \{\phi_1\}^1$$

and

$$K^{(2)} \{\phi_1\}^2 = \overline{\{\phi_1\}}^2 \qquad 2.23)$$

where $K^{(2)}$ is the new tower top deflection and $\{\phi_1\}^2$ is the new shape function. This new function is again substituted into 2.21) and the process is continued. As is shown in Ref. 2 this processes converges after a certain number of iterations N, so that

$$\lim_{N \to \infty} K^{(N)} = \frac{1}{W_1^2}$$

$$\text{and } \lim_{N \to \infty} \{\phi_1\}^N = \{\phi_1\}: \text{ exact modeshape.} \qquad 2.24)$$

In practice, only a finite number of iterations (about ten) are used to determine the first bending frequency W_1, and the modeshape. By repeating the process sufficiently, the modeshape approximation can be improved to any desired level of accuracy, i.e., after s cycles.

$$\overline{\{\phi_1\}}^s = \frac{1}{W_1^2} \{\phi_1\}^{s-1} \qquad 2.25)$$

when the iterative process has converged to this extent, the desired frequency is given by:

$$W_1 = \frac{1}{K^{(s)}} \qquad 2.26)$$

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A proof of the convergence of this method is given in Ref. 2. This proof also gives an indication of the manner in which the procedure can be used to evaluate higher modes as well. From the proof, it can be shown that if the assumed shape function contains no first mode contribution, the process will converge to the second bending mode. The means of eliminating the first-mode component from any assumed second modeshape is provided by the orthogonality condition:

$$[m] \{\phi_1\}^T \{\phi_2\} = 0 \qquad 2.27)$$

between the first and second modes. One uses this condition to eliminate any first mode contribution to the assumed second bending mode $\{\phi_2\}$. More details of this method are given in Appendix E.

All of the methods presented in this section are considered to be simple methods, compared to a huge finite element method such as SAPIV. It will be seen in Section 7.0 that for certain types of towers, these methods give inadequate results for the second bending frequencies. In such cases, larger and more expensive programs such as SAPIV are needed.

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3.0 TOWER TEST SPECIMENS

The objective of this chapter is to describe the physical properties and parameters of the towers tested at Rocky Flats. The vibration frequencies can then be estimated, once the proper parameters have been determined for use in the analysis methods just described. Four different towers are covered, two freestanding towers and two single-guyed towers.

3.1 A COR-TEN FREESTANDING TOWER

This tower (shown in Figure 3.1) has a height L of 55 ft, and is made of ASTM A-588 Steel (COR-TEN Steel). The pole has a dodecagonal cross section with an inside diameter at the tower top of 8.00 in. with a wall thickness of 0.3125 in. The pole diameter increases from top to bottom with a taper of 0.42 inches per foot, so that the inside diameter at the tower base is 31.10 inches. The inside diameter varies according to the equation:

$$d_i(x) = 31.100 - 0.035x, \quad 0 \leq x \leq 660 \qquad 3.1)$$

where x and d_i are in inches. The outside diameter of the tower varies according to the equation:

$$d_o(x) = 31.725 - 0.035x, \quad 0 \leq x \leq 660. \qquad 3.2)$$

To ease determination of the cross section area moment of inertia, the dodecagonal cross section is assumed to be circular. This assumption will be shown (Section 4.0) to give adequate frequency results, using the analysis methods of Section 2.0.

For a circular cross section, the area moment of inertia can be expressed as (Ref. 3):

$$I = \frac{\pi}{64} (d_o^4 - d_i^4)$$

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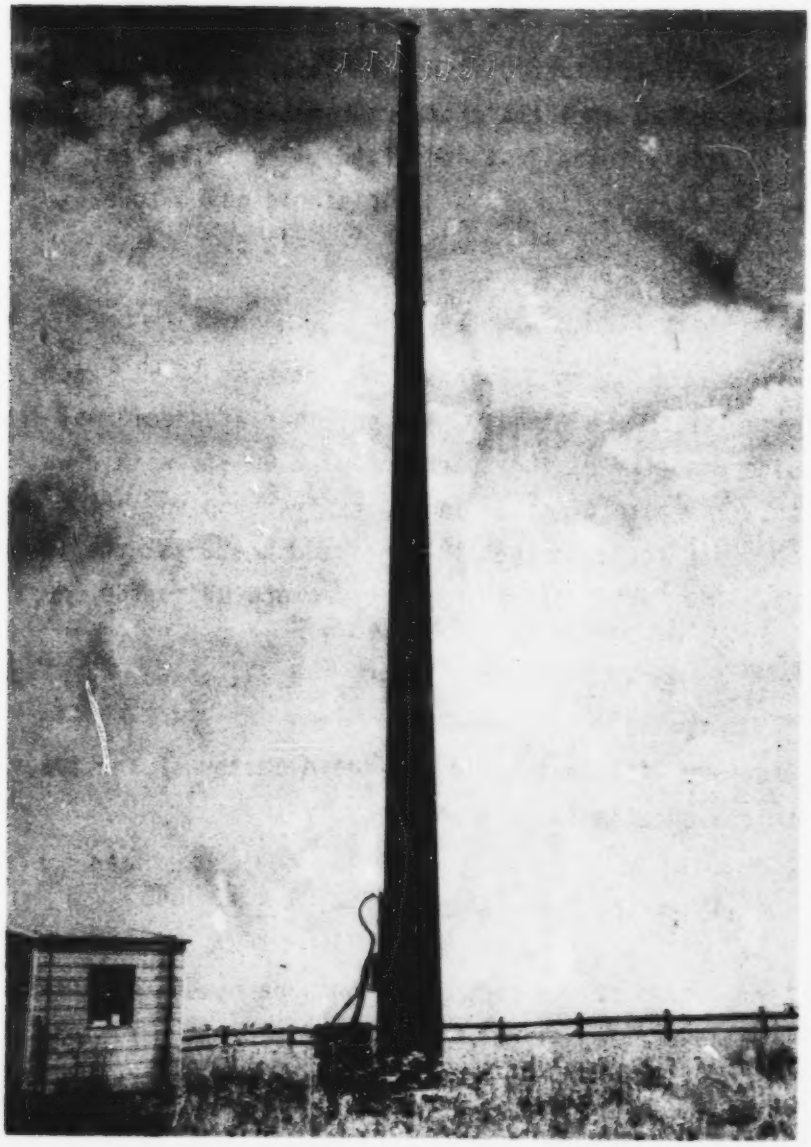

Figure 3.1: COR-TEN Freestanding Tower

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Using this formula for moment of inertia and the expressions for d_i and d_o 3.1) and 3.2) an expression for a dimensionless distribution of stiffness can be written in terms of $\xi = x/L$ as:

$$\frac{EI(\xi)}{EI_o} = f(\xi) = [-0.393\ \xi^3 + 1.617\ \xi^2 - 2.210\ \xi + 1.000] \quad \text{for } 0 \leq \xi \leq 1. \qquad 3.4)$$

where $EI_o = 1.141 \times 10^{\text{"}}$ lb/in.^{2} and $E = 30 \times 10^6$ lb/in.^{2}.

It should be noted that this dimensionless stiffness distribution corresponds to $f(\xi)$ in expression 2.12, page 14. A plot of this distribution is shown in Figure 3.2.

Besides the stiffness distribution $EI(x)$, the mass distribution $\rho A(x)$ must be found, where ρ is the density of the material and $A(x)$ is the material cross sectional area at a given tower height x. Since the density ρ is constant, for various tower heights, the area A may vary as a function of x. The area A may be expressed:

$$A = \frac{\pi}{4}\ (d_o^2 - d_i^2). \qquad 3.5)$$

Upon substitution of 3.1) and 3.2) into 3.5) the normalized expression for the mass distribution can be shown to be:

$$\frac{\rho A(x)}{\rho A_o} = h(\xi) = 1 - 0.737\ \xi,\ 0 \leq \xi \leq 1.$$

This function corresponds to the function $h(\xi)$ in expression 2.12 on page 14. As can be seen in Figure 3.3 this mass distribution is linear; ρA_o is the value of distributed mass at the tower base, equal to 0.023 lb-s^2/in.^{2}.

The fact that this tower consists of a single pipe has made the determination of the mass and stiffness distributions relatively easy, compared to

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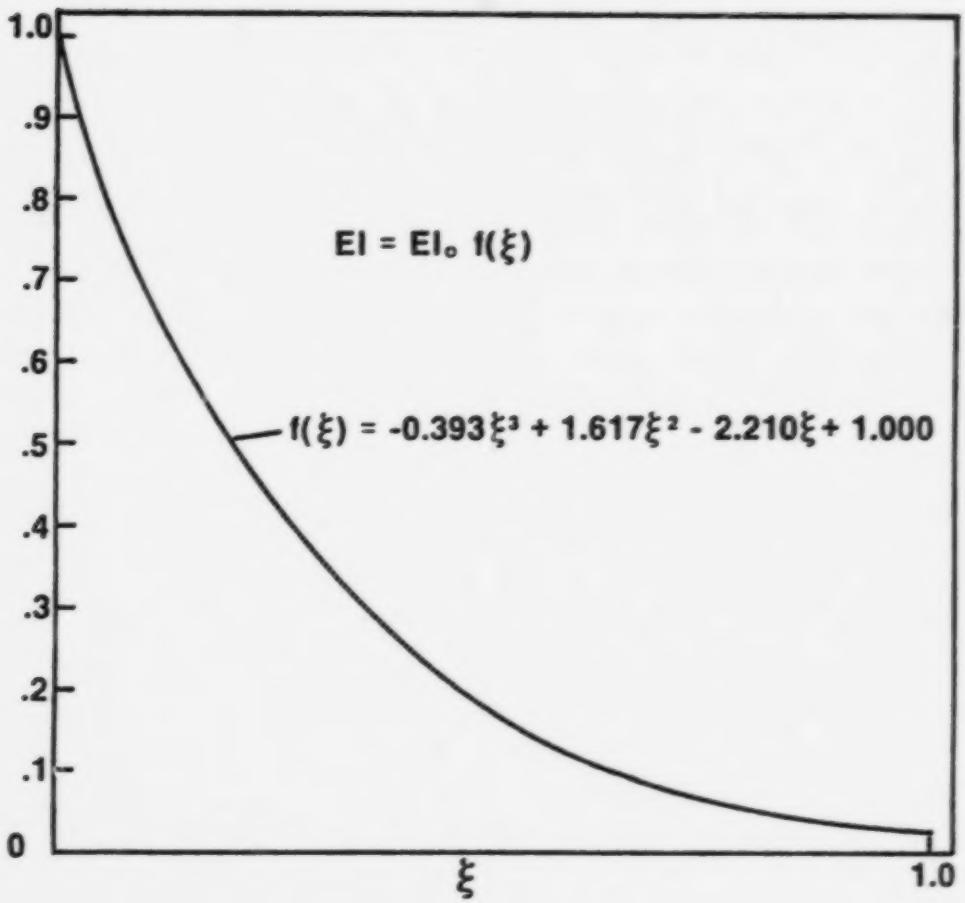

Figure 3.2: COR-TEN Dimensionless Stiffness Distribution f(ξ)

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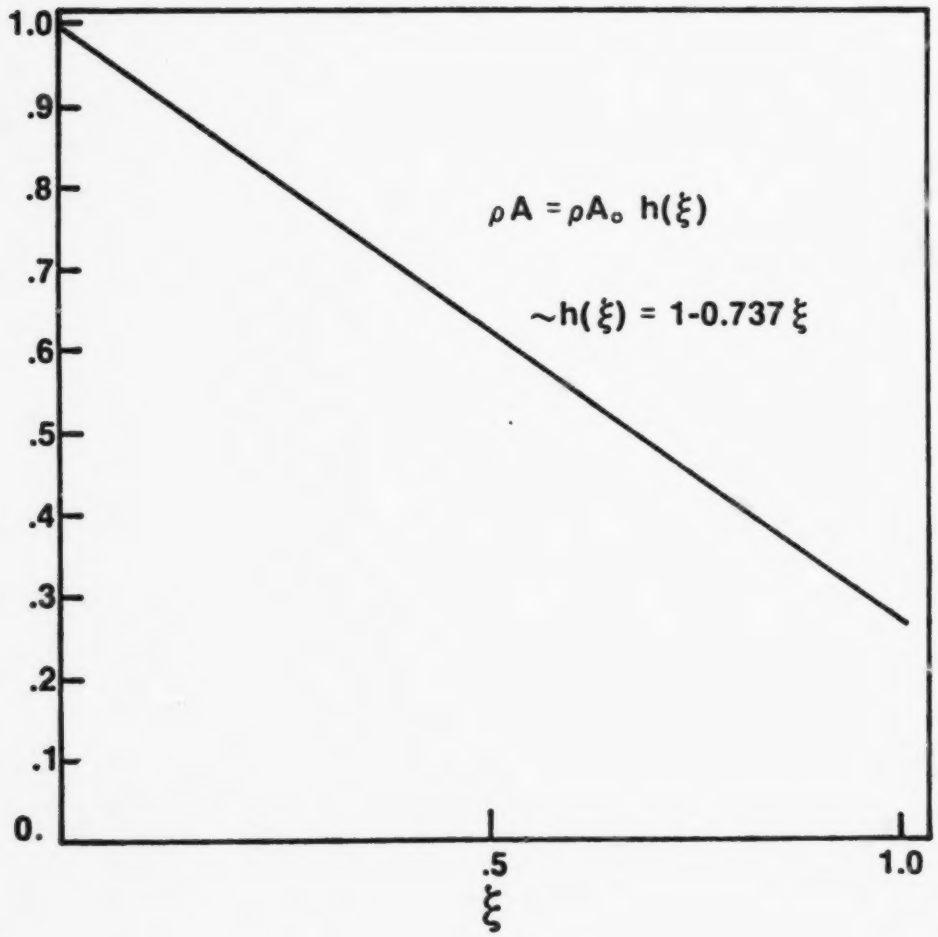

Figure 3.3: COR-TEN Dimensionless Mass Distribution $h(\xi)$

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a truss-type tower with many cross members. However as was described in Section 2.3 on the Rayleigh and Rayleigh-Ritz methods, a tower with a highly tapered stiffness distribution, such as this COR-TEN tower, can pose greater difficulty than a tower with uniform stiffness distribution, for accurate determination of bending frequencies. This tower has a slightly tapered mass distribution and a highly tapered nonlinear stiffness distribution. The analysis of this tower, using the simple methods, will be described in Section 5.0.

A second freestanding tower will now be described. Unlike the COR-TEN tower, this tower is a truss tower with many cross members.

3.2 A ROHN SSV FREESTANDING TOWER

This truss tower is shown in Figure 3.4 and has a height L of 60 ft and is composed of three 20-ft sections. The members of each section are composed of galvanized steel with a modulus of elasticity (E) of 30×10^6 lb/in.^{2}.

As can be seen in the picture, this tower has three legs with members which crisscross from one leg to the other. The distance between tower legs decreases with tower height, but the length and cross sectional area of each leg is the same in each of the three sections.

For each of the simple analysis methods described in Section 2.0, the tower moment of inertia will be determined by considering the effects of the three legs only; the effects of the cross members will be neglected in order to simplify the procedure. In order to calculate the moment of inertia at a certain height x, it should be noted that the distance between the center of each leg is the same at that height, so that the legs form an equilateral triangle, as shown in Figure 3.5.

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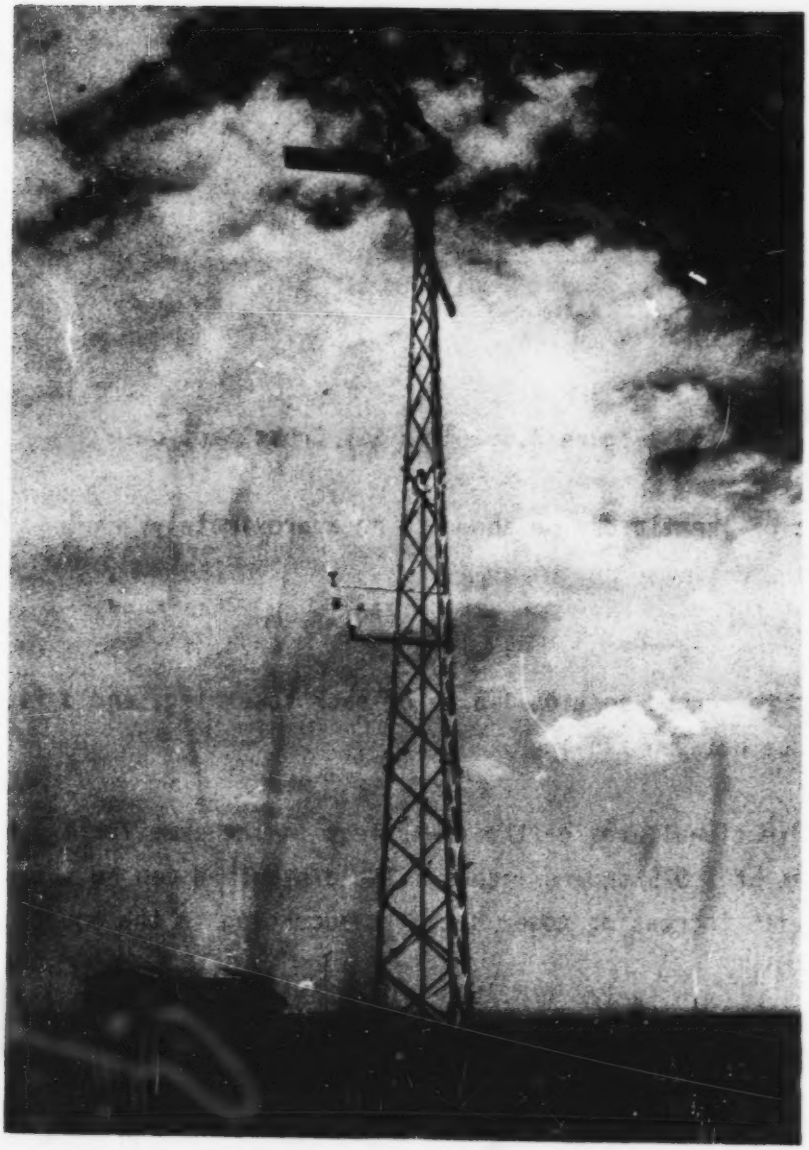

Figure 3.4: Rohn SSV Tower

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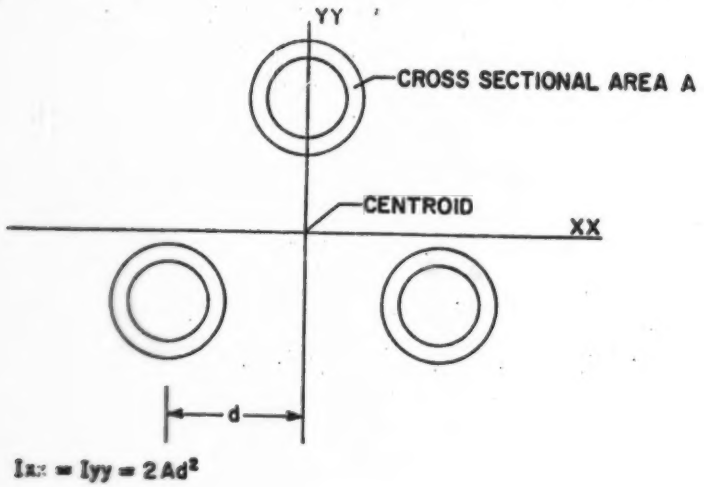

Figure 3.5: Rohn SSV Cross Section

The moment of inertia can be shown to be approximately:

$$I \approx 2Ad^2,$$

where A is the cross sectional area of each tower leg, and d is one-half of the distance between two legs.

The cross sectional area of the tower legs is the same for any height x and has a value of 1.075 square inches. The distance d can be shown to decrease with increasing tower height x according to the equation:

$$d(x) = \begin{bmatrix} 1/2\ (78.7500 - 0.1016x),\ 0 \leq x \leq 480 \\ 1/2\ (66.0000 - 0.0750x),\ 480 \leq x \leq 720 \end{bmatrix} \qquad 3.7)$$

Substitution of this expression for d into the expression for I, given above, and using the dimensionless variable

$$\xi = x/L$$

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gives:

$$\frac{EI(\xi)}{EI_o} = \begin{bmatrix} (1.000 - 1.872\ \xi + 0.861\ \xi^2)\ 0 \leq \xi \leq 0.667 \\ (0.702 - 1.163\ \xi + 0.468\ \xi^2)\ 0.667 \leq \xi \leq 1.000 \end{bmatrix} \qquad 3.8)$$

where: $A = 1.075\ in.^2$

$E = 30 \times 10^6\ lb/in^2$

$L = 720\ in.$

and $EI_o = 1.000 \times 10^{11}\ lb/in^2$

As can be seen in Figure 3.6, the stiffness distribution decreases from base to the tower top; the effects of the cross members having been neglected. Also, the stiffness distribution is nonlinear and obeys the above expression.

The mass distribution has been approximated by taking the total mass of each section (including the mass of cross members) and dividing by the section length. For the three sections the mass distributions are:

bottom section: $\rho A_0 = 0.0071\ lb\text{-}s^2/in.^2$

middle section: $\rho A_1 = 0.0052\ lb\text{-}s^2/in.^2$

top section: $\rho A_2 = 0.0051\ lb\text{-}s^2/in.^2$

As can be seen, the mass distribution for each section is assumed to be simply a constant, and the total tower mass distribution has discontinuities at the points where one section begins and one ends.

Finally, this tower was assumed to have a top plate of 50 pounds. The tower was not tested with a machine on top. In two of the methods of Section 2.0, a tip mass ratio μ was defined as:

$$\mu = \frac{M}{\rho A_o L} \qquad 3.9)$$

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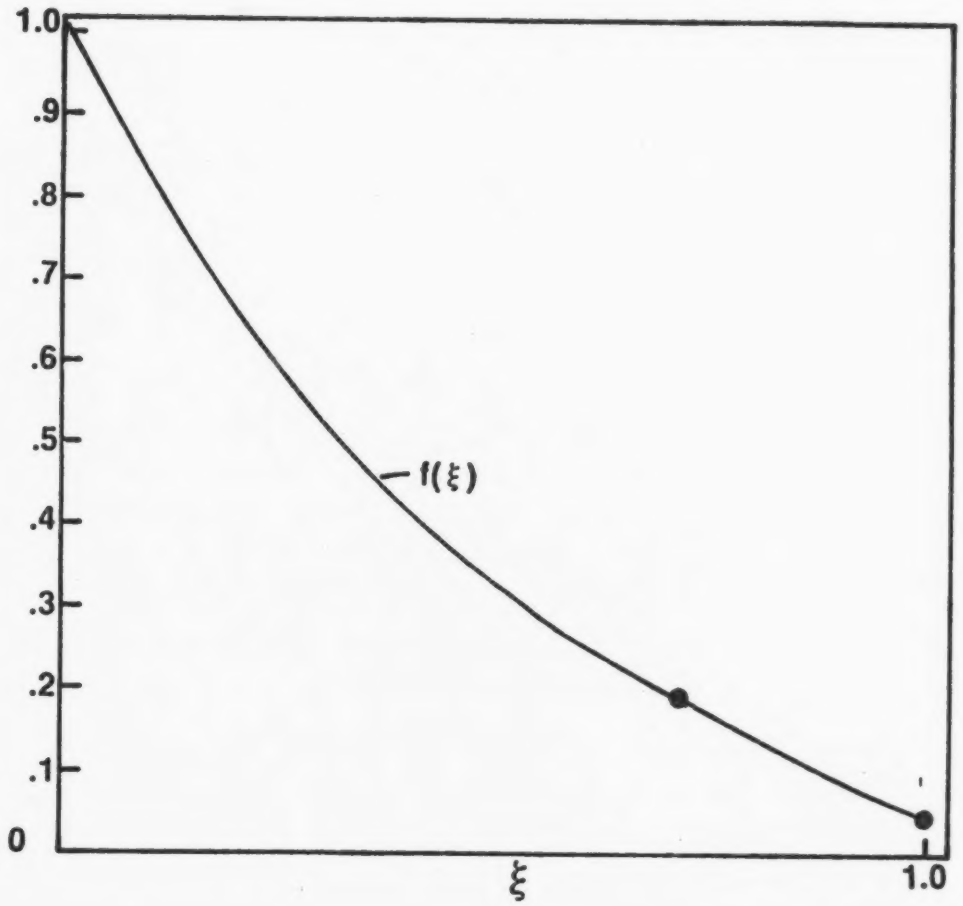

Figure 3.6: Rohn SSV Dimensionless Stiffness Distribution f(ξ)

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where M is the mass of the top weight, ρA_0 is the tower base mass distribution, and L is the tower height. For the value of top mass of 0.1294 lb-s^2/in., tower base mass distribution of 0.0071 lb-s^2/$in.^2$, and tower height L of 720 in., a value of $\mu = 0.025$ is obtained.

The parameters and properties for input data for the analytical methods have now been described for this tower. It remains to be seen whether the simple methods in which this tower is modeled as a cantilever beam give adequate results compared to test results, and whether neglect of the cross members in the stiffness distribution calculation is justified.

3.3 A COR-TEN SINGLE GUYED TOWER

Figure 3.7 shows this COR-TEN single guyed tower. The tower has four guy wires attached at the same level, spaced 90° apart.

This tower has a uniform (constant) mass and stiffness distribution. The steel pipe has an outside diameter of 16 in. and a wall thickness of 3/8 in. These properties are constant for the entire tower length. The tower moment of inertia can be expressed:

$$I = \frac{\pi}{64} (d_o^4 - d_i^4) \qquad 3.10)$$

where d_o is the outside pipe diameter and d_i is the inside diameter. For this pipe:

$$d_o = 16.00 \text{ inches}$$
$$\text{and } d_i = 15.25 \text{ inches},$$

giving a value of 562.1 $in.^4$ for I. Then EI is equal to 1.686×10^{10} lbf/$in.^2$, where E is equal to 30×10^6 lbf/$in.^2$. The mass distribution ρA can be found by first determining the cross sectional area A of the pipe:

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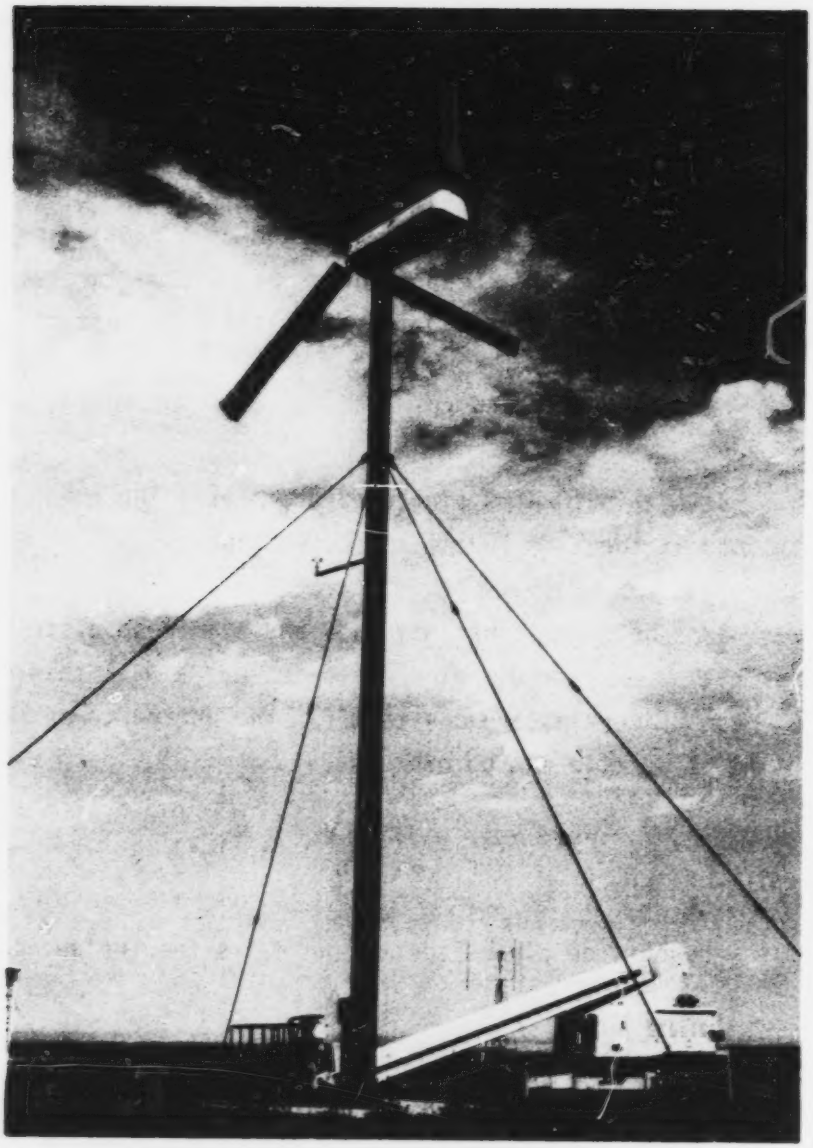

Figure 3.7: COR-TEN Single Guyed Tower

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$$A = \frac{\pi}{4} (d_o^2 - d_i^2). \qquad 3.11)$$

giving a value of A = 18.41 in.^{2}. The density ρ of the steel is 0.00073 lb-s^2/in.^{4}. The mass distribution of this tower is then:

$$\rho A = 0.0134 \frac{\text{lb-s}^2}{\text{in.}^2}.$$

This tower was tested with a machine having a weight of 2,589 lb at the tower top. The top mass ratio μ can be found from:

$$\mu = \frac{M}{\rho A_o L}.$$

The height L of this tower is 660 inches and

$$M = 6.70 \frac{\text{lbf-s}^2}{\text{in.}}.$$

The value for μ is then 0.758.

Now that the tower properties alone have been determined, the guy wire properties need to be defined. It can be shown that for four guy wires spaced 90° apart the guy stiffness coefficient, as defined in Section 2.4, is:

$$K_c = 2 \left(\frac{A'E'}{\ell}\right) \cos^2 \theta \qquad 3.12)$$

Each of these guy wires are actually steel rods with a cross sectional area A' of 0.47 in.^{2}, an elastic modulus E' of 30 x 10^6 lbf/in.^{2}, and a length ℓ of 679 in. The angle that the guy wire makes with the ground (θ) is 45°. K_c is thus calculated to be 20,770 lbf/in. The guy attachment level a is 480 in.

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In the Rayleigh-Ritz procedure described in Section 2.3 a dimensionless guy wire stiffness coefficient and attachment level were defined as:

$$\bar{K}_c = \frac{K_c L^3}{EI_o} \qquad 3.13)$$

and $\xi_a = a/L$,

where EI_o is the tower base stiffness, L the tower height, and a the guy wire attachment level. For this tower EI_o is 1.686×10^{10} lb/in.^{2}, and

$$\bar{K}_c = 354.1,$$
$$\xi_a = 0.73.$$

All of the properties and parameters for data input to the simple methods have now been described. In each of these methods the guy wire effects are handled by adding an extra term to the tower potential energy. The effects of the guys are to increase the natural frequencies of the system, because the stiffness has been increased. The simple formula for the added potential energy term given in 3.12) is only valid when the guy wires have been sufficiently tensioned so that there is no coupling between the guy wires and the tower. This criterion will be satisfied only if the frequency of the fundamental mode of each guy wire is well separated from the fundamental tower frequency (with the guy wires attached). The effects of the guy wires will be explored more completely in Section 6.0.

As will be seen in Section 6.0, uncertainties in the measurement of guy wire properties such as the cross sectional area A' and elastic modulus E' can greatly influence the accuracy of the tower frequencies. In the next tower to be presented, accurate values of guy wire A' and E' values are harder to determine.

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3.4 A ROHN 25G GUYED TOWER

Figure 3.8 shows a Rohn 25G single guyed tower. The tower has three guy cables attached at the same level, spaced 120° apart. The mass and stiffness distributions of this tower are uniform, with the values:

$$EI = 3.14 \times 10^6 \text{ lb-ft}^2$$
$$\text{and } \rho A = 0.124 \text{ lb-s}^2/\text{ft}^2.$$

The tower stiffness distribution was determined by first finding the moment of inertia of a tower cross section, as shown in Figure 3.5 for the Rohn SSV tower.

Because the distance between the tower legs remains constant for various heights, the moment of inertia, and thus the stiffness distribution is a constant (unlike the Rohn SSV tower). The mass distribution is also constant, since the weight and length of each tower section is the same. The effects of the cross braces were neglected in the determination of the stiffness distribution but included in the weight and mass distribution.

As shown in Figure 3.8, the guy wires are attached to the 40 ft tower at a height of 35 ft. The length of each guy cable is 48.1 ft, and the angle that the guy cable makes with the ground is 46.7°.

Unlike the previous tower in which the guy cables were solid rods, these guy cables are composed of seven wire strands, as shown in Figure 3.9. Each wire rope has a diameter of 3/16 in.

The cross sectional area of this wire rope is found by adding the area of each strand. The diameter of one strand is one-third of the diameter of the entire wire rope:

$$d = 1/16 \text{ in. : one strand}$$

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The area A' of the guy cable is then found to be 0.00015 ft^2. Errors in the measurement of the diameter of the guy cable can have an effect on the natural frequencies of the tower. For example, if the error in the measurement of the total cable diameter is 1/32 in., or about 17 percent, the error in the total area calculation could be as much as 36 percent. This error in A' will also cause the guy wire stiffness coefficient K_c to be in error by about 36 percent, which will cause large frequency errors.

Another source of error is in the estimation of the guy wire elastic modulus E'. For the COR-TEN guyed tower the guy wires were solid steel rods, and the elastic modulus was easily determined to be 30 x 10^6 $lb/in.^2$. For the guy cables on the Rohn 25G, which are not solid steel rods, but are composed of strands, the elastic modulus is harder to determine.

From the Machinery's Handbook (Ref. 11) the elastic modulus of this extra heavy strength, seven-strand wire rope is estimated to be 20 x 10^6 $lb/in.^2$. There exists the possibility that this value for E' can change with time, as temperature changes occur, or the strands become more packed together. Precise values for the E' are not known.

Two guyed towers have now been described. The analysis of guyed towers can become complicated if the guy fundamental frequencies are not well separated from the tower fundamental frequency (with the guy wires attached). The guy wires must be tensioned sufficiently so that guy wire-tower coupling will not occur. Another source of difficulty occurs when precise values of cross sectional area A' and elastic modulus E' cannot be determined.

Now that the tower test specimens have been described, the test procedures and test results will be explained. The test results should provide an excellent means of evaluating the accuracy of the simple methods.

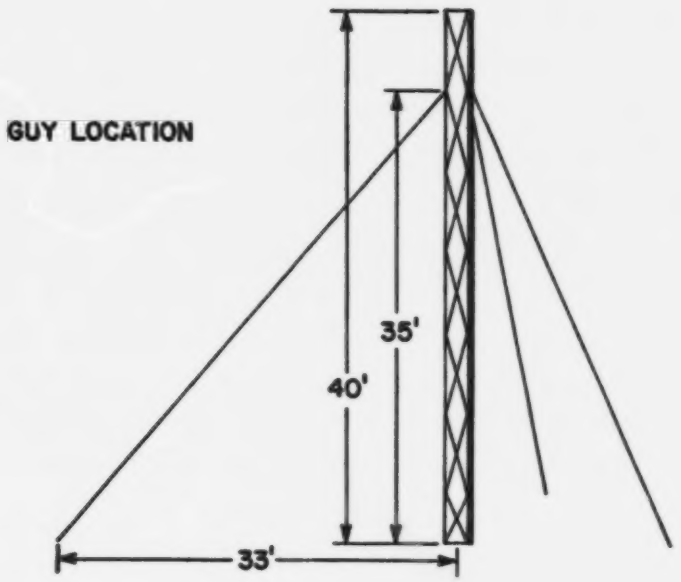

Figure 3.8: Rohn 25G Single Guyed Tower

Figure 3.9: Seven Strand Wire Rope

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4.0 TEST METHODS AND TEST RESULTS

A technique currently being used at the Rocky Flats Test Center for dynamic testing is commonly known as impact testing. The structure can be excited with an impulse, using a hammer and an attached load cell (9), as shown in Figure 4.1. The input force can be accurately measured so that the structure (10) is excited with a known input.

The response of the structure can be measured with an accelerometer (8), as shown in Figure 4.1. Provided that the input and response signals are Fourier transformable (Ref. 4), the frequency response function can be computed. The input and response data can be readily reduced with the aid of a spectrum analyzer, such as the Hewlett Packard 5420A.

Once the input and output signals have been accurately measured, the frequencies of vibration of the structure can be determined by finding the transfer function and the frequency response function.

4.1 ROHN 25G TOWER TEST RESULTS

The Rohn 25G was tested without a wind turbine on top, but with a top plate of 380 lbs on the tower top. Table III shows the test results, using the impact testing methods. In this case, the three guy wires were tensioned sufficiently so that each had nearly the same fundamental frequency. As can be seen from Table III, the guy fundamental frequency was well separated from the tower first bending frequency, so that resonance between the guy wires and tower did not occur.

TABLE III: ROHN 25G TEST RESULTS

First Mode Bending	2.6 Hz
First Mode Torsional	5.7 Hz
Second Mode Bending	7.8 Hz
Guy Wire Fundamental	6.9 Hz

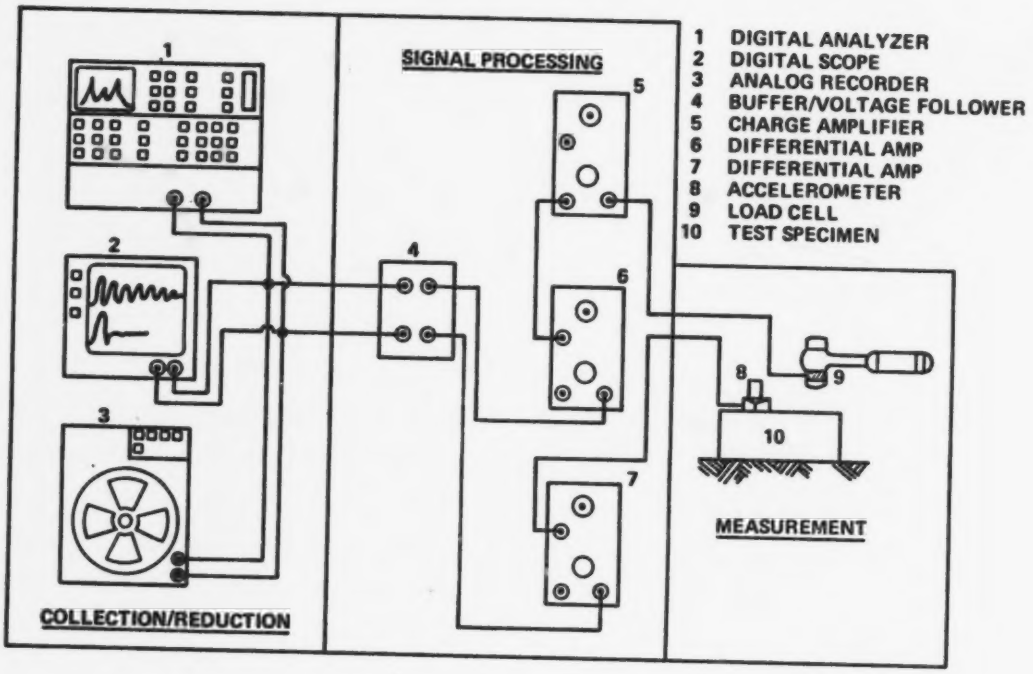

Figure 4.1: Test Setup

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A transfer function, obtained by an impact test, is shown for the Rohn 25G in Figure 4.2. For this test, the accelerometer was placed at the tower top. The tower was excited with the hammer at the tower top in such a way that both the first bending and first torsional frequencies were excited. The first bending frequency was measured as 2.64 Hz and the first torsional frequency as 5.66 Hz. A second transfer function, showing the second bending mode, is shown in Figure 4.3. From this figure it can be seen that the second bending frequency is 7.83 Hz.

4.2 ROHN SSV TOWER TEST RESULTS

The Rohn SSV was tested without a machine on top but with a top plate which weighs 50 pounds. For this tower the first bending frequency was measured as 3.3 Hz. The second bending frequency was measured as 12.7 Hz. Similar test methods were used to obtain transfer functions for this tower, as for the Rohn 25G tower.

Accurate estimation of the first and second bending frequencies for this tower is difficult, especially with the simple hand analysis methods. As shown in Section 2.0, the simple Rayleigh hand analysis and Rayleigh-Ritz hand analysis give inaccurate results for towers in which the stiffness distributions taper to nearly a point at the tower top. Also this tower has many cross members in which the effects are not accounted for in the determination of the stiffness distribution. For this reason, a second freestanding tower at Rocky Flats was tested: the COR-TEN freestanding tower, which has no cross members. This tower does have a highly tapered stiffness distribution however, similar to the Rohn SSV.

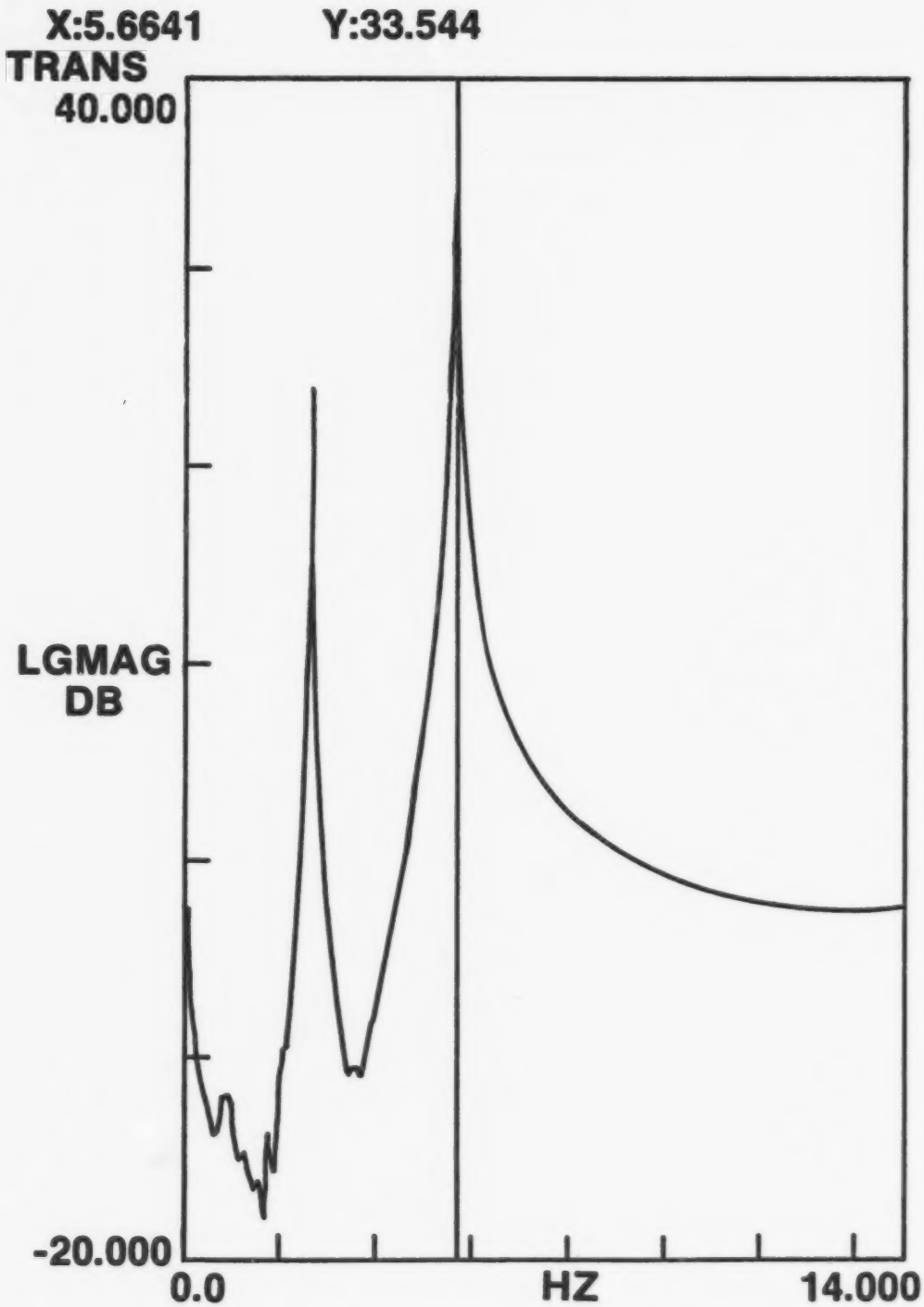

Figure 4.2: Rohn 25G First Mode Bending and First Mode Torsional

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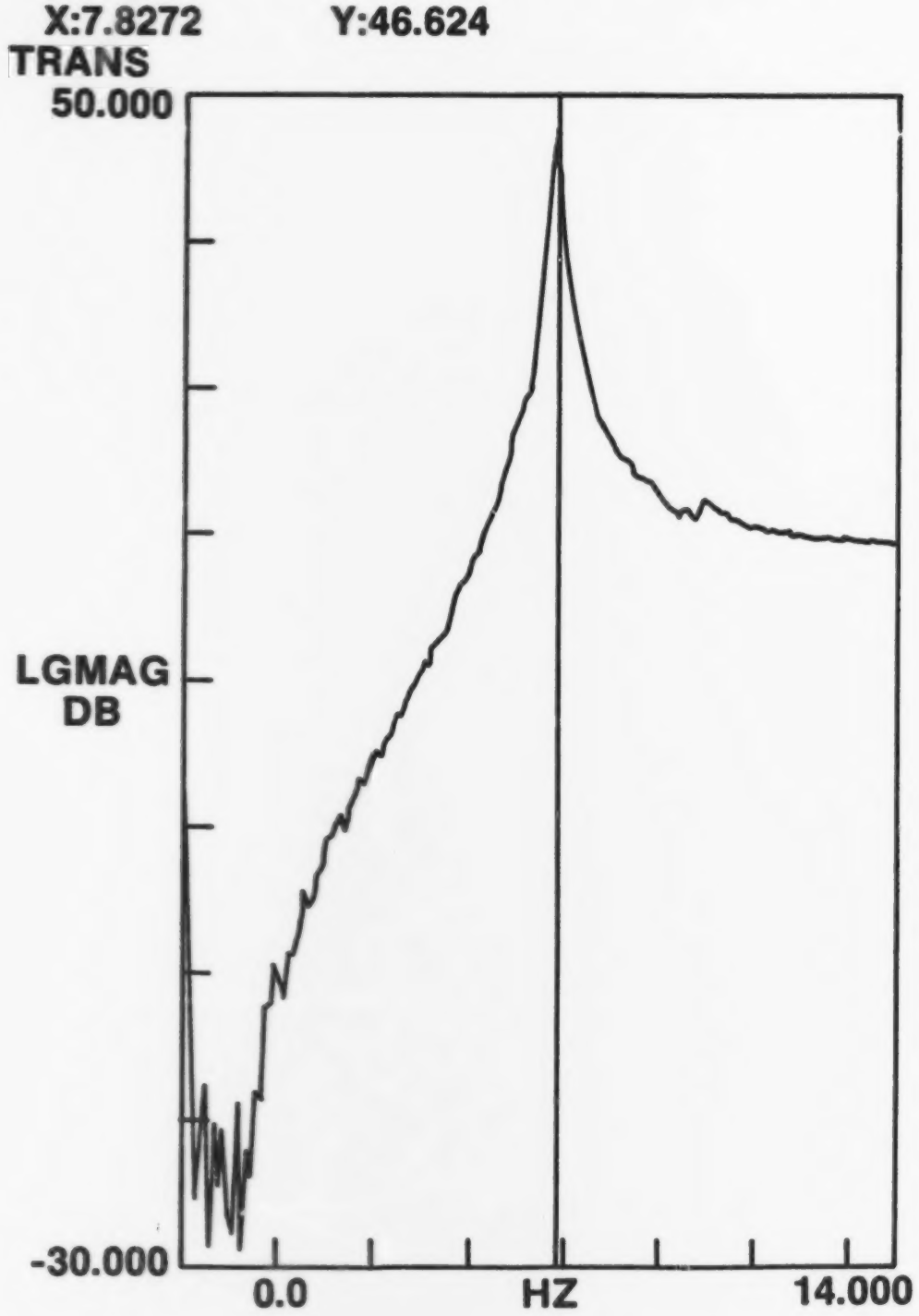

Figure 4.3: Rohn 25G Second Mode Bending

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4.3 COR-TEN FREESTANDING TOWER TEST RESULTS

The COR-TEN freestanding tower, which was described in Section 3.0, was tested without a machine on top but with approximately a 50-pound top plate. The first bending frequency was measured as 3.1 Hz and the second bending frequency as 12.7 Hz.

The highly tapered stiffness distribution of this tower causes some difficulty in the use of the simple methods for accurate frequency determination. The fact that this tower is a pole, without any cross members, ensures better results than the Rohn SSV. The stiffness distribution is much easier to determine for this tower than for the Rohn SSV.

In the analysis of guyed towers, the guy wire fundamental frequencies need to be determined. To illustrate the importance of guy wire dynamic effects, a COR-TEN guyed tower was tested.

4.4 COR-TEN GUYED TOWER TEST RESULTS

For the Rohn 25G tower described previously, the guy wires were tensioned sufficiently so that each had nearly the same fundamental frequency. Also, the guy fundamental frequencies were well separated from, and higher than the tower fundamental frequency. With these criteria satisfied, very little coupling occurs between the tower and guy wires, and the guy wire effects can be modeled using the simple expression given by 2.10 in Section 2.3.

For the COR-TEN tower however, the guy wire fundamental frequencies were probably not well separated from the tower fundamental frequency.

The first fundamental tower frequency was measured as 2.4 Hz, as can be seen from Figure 4.4. The guy wires each had nearly the same fundamental frequencies believed to be at about 2.5 Hz.

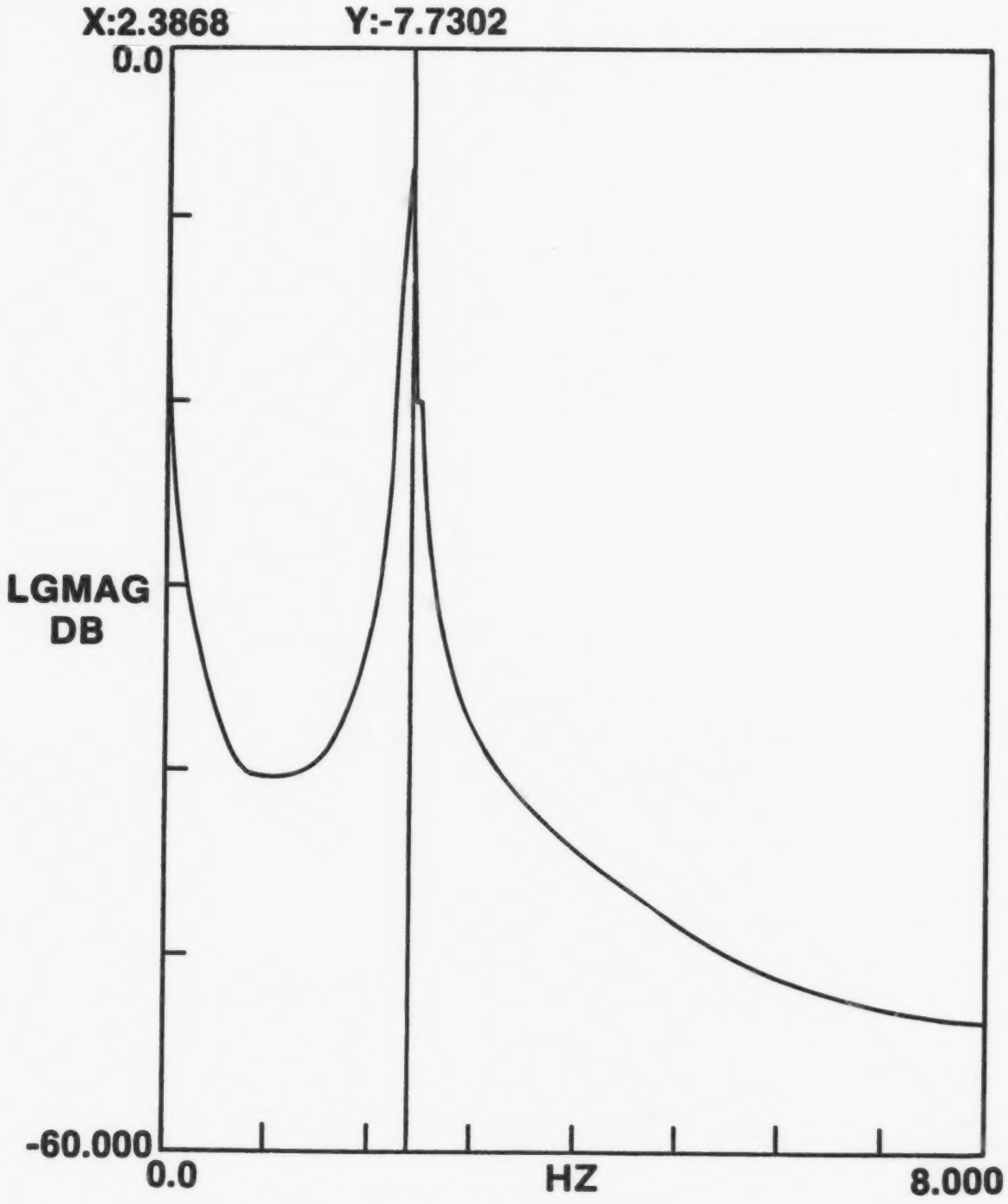

Figure 4.4: COR-TEN Guyed Tower First Mode Bending

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As will be shown in Section 7.0, the use of the simple methods for this tower give inaccurate results compared to test results. Guy wire-tower coupling is not modeled with these methods. It will be shown that the simple methods give results higher than the actual test results. The possible reasons for this error will be further described in Section 7.0, in which the analytical results are compared to test results.

It must be emphasized that these tower test results are not perfect. These measurements are accurate only to a degree dependent on the limitations of the instrumentation, testing techniques and experimental test set up. Although the errors in these measurements are not exactly known, the good correlation between test results and analytical results (to be seen later) for a number of cases gives confidence in the testing techniques.

In the next section, the use of each method will be described for the COR-TEN freestanding tower. Particular attention will be focused on the correct data input for a tower. This section will mainly be presented to give the designer or analyst a feel for the correct use and accuracy of the methods, and correct data input.

5.0 USE OF THE METHODS

Now that the tower test specimens, test methods and test results have been described, the analysis of one tower with each method will be shown. Special emphasis will be placed on the correct parameter and data input for the methods and computer codes. A description of the reasons for inaccuracies in analysis results compared to test results will also be given. A summary of results for other towers will be given in Section 7.0.

Analysis of the COR-TEN Freestanding Tower

Because the COR-TEN is a freestanding tower, the Rayleigh method is a good candidate for a rough estimate of the first bending frequency. In particular, the simple Rayleigh hand method of Section 2.2 will first be used.

Simple Rayleigh Hand Method

Equation 2.7 (Section 2.2) was developed for towers with linear mass and stiffness distributions of the form:

$$EI(x) = EI_0 (1 - \beta x/L)$$
$$\rho A(x) = \rho A_0 (1 - \gamma x/L), \qquad 5.1)$$

The nonlinear stiffness distribution will first be approximated by a linear distribution of the form given by 5.1.

The value for β can be found from:

$$\beta = 1 - \frac{EI\ (\xi = 1)}{EI_0},$$

where EI (ξ = 1) is the stiffness at the tower top. In the case of the COR-TEN tower the value for β is 0.986. The value of γ for the linear mass distribution is 0.737, as given in section 3.1. The values of stiffness

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and mass distribution at the tower base are given in Section 3.1. The stiffness distribution is thus approximated, using this technique, by the linear distribution as shown in Figure 5.1. It should be noted that the actual total tower stiffness (area under the curve) is overestimated by this straight line approximation. The resulting frequency estimate will thus be too high.

The tip mass ratio μ was defined as:

$$\mu = M/\rho A_0 L.$$

In this case a weight of 50 lb was located at the tower top, giving a value of μ equal to 0.01. Using these parameter values in Equation 2.7) gives a frequency value of 3.95 Hz., compared to the test value of 3.1 Hz. The error in this case is seen to be about 30 percent.

A more accurate value might be obtained by substituting the nonlinear stiffness distribution $f(\xi)$ given by 3.4) of Section 3.1 into the Rayleigh quotient:

$$P_1^2 = \frac{EI_0}{\rho A_0 L^4} \quad \frac{\int_0^1 f(\xi)\,(\phi'')^2\,d\xi}{\int_0^1 h(\xi)\,\phi^2\,d\xi + \mu\phi^2(1)} \qquad 5.2)$$

and using the same assumed first bending modeshape

$$\bar{\phi}(\xi) = \xi^4 - 4\xi^3 + 6\xi^2. \qquad 5.3)$$

When the above details have been carried out, the first bending frequency result is 3.62 Hz. giving an error of about 16 percent. These calculations show that accurate input of the stiffness distribution is important for accurate frequency results. In addition, the assumed modeshape should

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approximate the exact modeshape. It is probable that the 16 percent error in the second result given above is due to modeshape error. In Figure 5.2 a comparison of the assumed modeshape (Equation 5.3) with a more accurate modeshape determined by use of the MITGUY computer code (to be described later) can be seen.

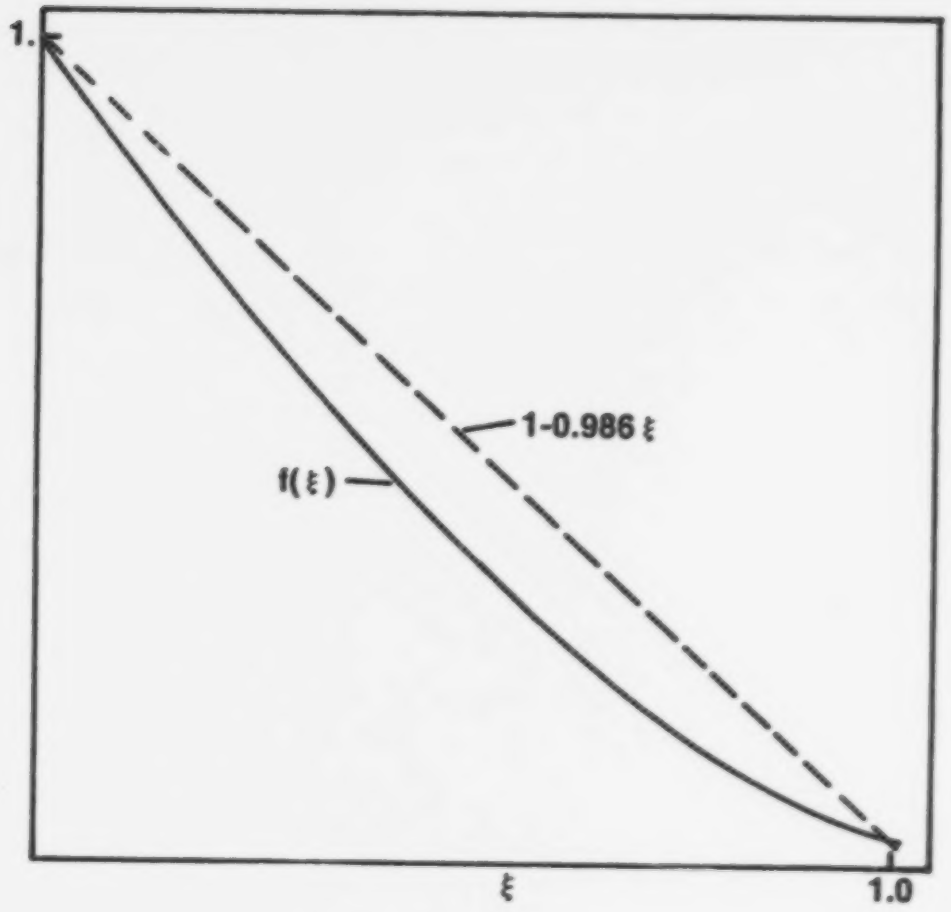

Figure 5.1: Linear Approximation of the Stiffness Distribution

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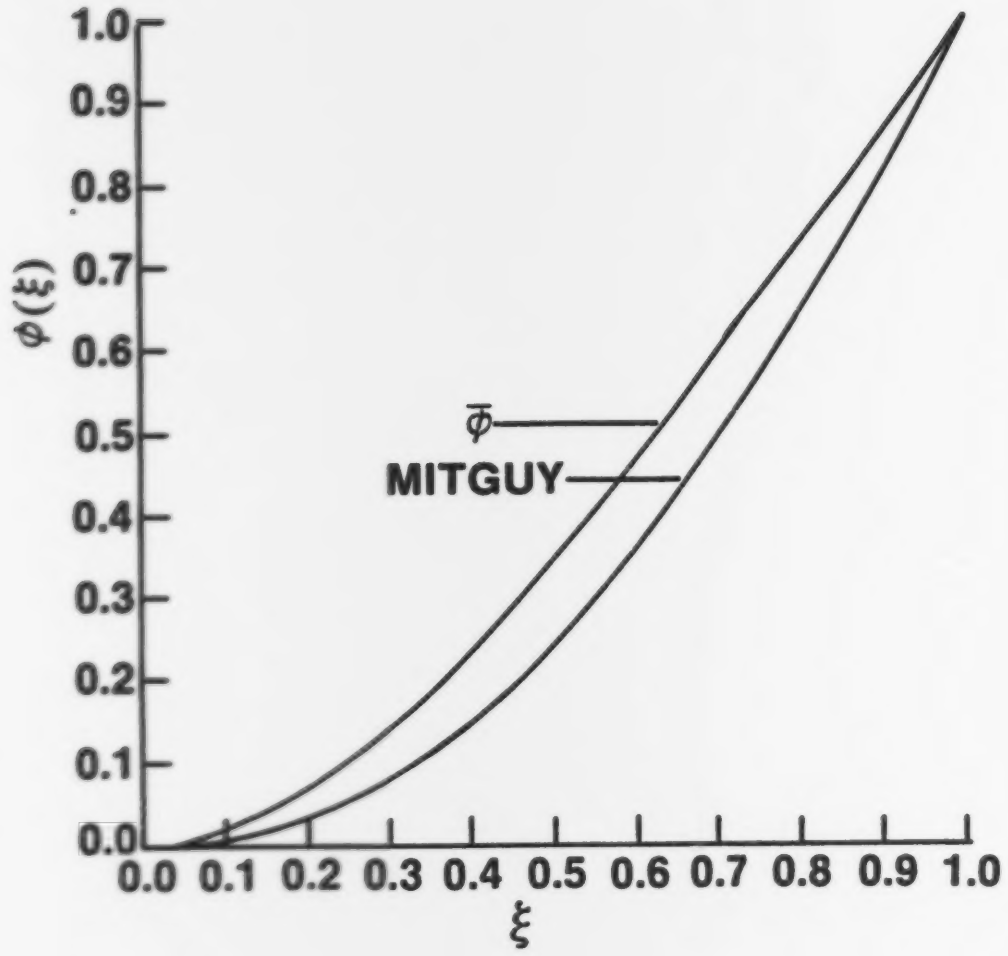

Figure 5.2: Modeshape Comparisons: Results For The COR-TEN From MITGUY Computer Code Compared to the Polynomial $\bar{\phi}(\xi)$.

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A more accurate value for the frequency estimate might be attained by using a more accurate modeshape approximation. As was mentioned in the section on the Rayleigh method (Section 2.2) the resulting frequency estimates are higher than the exact frequencies. A more accurate modeshape approximation will then give lower frequency values, closer to the exact values. The analyst could try several assumed shape functions which approximate a first bending modeshape and which satisfy at least the tower base boundary conditions. The best modeshape approximation would then be the one which gives the lowest frequency estimate.

Analysis Using the Rayleigh Computer Code

As explained in Section 2.2 this code can be used to analyze towers with nonlinear mass and stiffness distributions. The tower can be divided into N segments and constant values of mass and stiffness input along each segment. In addition, the user can input any desired modeshape. Appendix A gives a listing of the program as well as a printout of the two data files "TWPROP" and "MODES," both used by the program for data input.

The program uses one of four assumed mode shapes from the file "MODES" and reads the beam mass and stiffness properties from file "TWPROP." In addition the user has the ability to input a nonstandard modeshape directly from the terminal, rather than an assumed modeshape from file "MODES."

As has been noticed previously, the resulting frequency estimate depends upon how accurately the true tower stiffness and mass distributions are input into the analysis method, as well as the accuracy of the assumed modeshaped. For this COR-TEN tower, two cases were run: Case 1 consisted of use of the modeshape approximation given by 5.3) and division of the tower into six elements. Case 2 consisted of use of the same modeshape but division of the tower into twenty elements. Table IV shows the resulting frequency estimates for each case.

TABLE IV: FREQUENCY RESULTS FROM
THE PROGRAM RAYLEIGH.

Case 1: Modeshape Approximation ϕ With Six Elements

$p = 4.2$ Hz.

Case 2: Modeshape Approximation ϕ With Twenty Elements

$p = 3.65$ Hz.

It must be emphasized that accurate input of the stiffness and mass distributions, as well as the modeshape approximation is important in obtaining accurate frequency estimates. For this COR-TEN tower the stiffness distribution decreases radically from base to height. It is necessary to use many elements for accurate stiffness distribution representation, so that good frequency estimates can be obtained.

Once the stiffness and mass distributions have been input as accurately as possible, one then needs to select an accurate modeshape approximation. One can experiment with various shape functions until the lowest frequency estimate is obtained. Often the search for an adequate shape function can become very tedious. The frequency result depends heavily on the accuracy of this single shape function.

In the Rayleigh-Ritz procedure a linear combination of assumed shape functions is used as a modeshape approximation. The frequency results will be more accurate for the COR-TEN tower, because a linear combination of shape functions is being used instead of just one assumed shape. The simple Rayleigh-Ritz hand analysis will now be used, to determine the first two bending frequency estimates.

Simple Rayleigh-Ritz Hand Analysis

First, this method will be used by assuming the mass and stiffness distributions to be linear. The k_{ij} and m_{ij} coefficients can then be expressed:

$$\begin{aligned} k_{11} &= 28.800 - 4.800\beta \\ k_{12} &= 24.000 - 6.857\beta \\ k_{22} &= 34.286 - 12.857\beta \\ m_{11} &= 2.311 - 1.854\gamma + 9.00\mu \\ m_{12} &= 2.070 - 1.700\gamma + 9.00\mu \\ \text{and } m_{22} &= 1.882 - 1.573\gamma + 9.00\mu. \end{aligned} \qquad 5.4)$$

With the values of $\beta = 0.986$, $\gamma = 0.737$ and $\mu = 0.01$, as given previously, the values for k_{ij} and m_{ij} are:

$$\begin{aligned} k_{11} &= 24.067 & m_{11} &= 1.035 \\ k_{12} &= 17.239 & m_{12} &= 0.907 \\ k_{22} &= 21.609 & m_{22} &= 0.813. \end{aligned} \qquad 5.5)$$

Then as given in Section 2.3:

$$\begin{aligned} g &= 0.019 \\ q &= 10.660 \\ 1 &= 222.881. \end{aligned} \qquad 5.6)$$

The polynomial:

$$0.019\ (W^2)^2 - 10.660\ W^2 + 222.881 \qquad 5.7)$$

must then be solved for the two values of W giving:

$$W_1 = 4.66 \text{ and } W_2 = 23.22$$

The frequency estimates are obtained by multiplying W_1 and W_2 by:

$$\sqrt{\frac{EI_o}{\rho A_o L^4}} = 5.11/\text{sec.}$$

giving $p_1 = 3.8$ Hz and $p_2 = 18.88$ Hz, compared to test results of $p_1 = 3.1$ Hz and $p_2 = 12.7$ Hz.

It can be seen from the above results that the use of the linear stiffness distribution ($\beta = 0.986$) results in large errors in the frequency results. Instead of using a linear stiffness distribution the actual nonlinear distribution given by:

$$f(\xi) = -0.393\ \xi^3 + 1.617\ \xi^2 - 2.210\ \xi + 1.000 \qquad 5.8)$$

will be used to compute the dimensionless stiffness coefficients:

$$k_{ij} = \int_0^1 f(\xi)\ \psi_i''(\xi)\ \psi_j''(\xi)\ d\xi \qquad 5.9)$$

as developed in Section 2.3.

After this nonlinear distribution has been substituted into 5.9) and the same two polynomials:

$$\psi_1 = \xi^4 - 4\xi^3 + 6\xi^2$$
$$\psi_2 = 3\xi^5 - 10\xi^4 + 10\xi^3 \qquad 5.10)$$

have been used, the k_{ij} coefficients are:

$$k_{11} = 20.208$$
$$k_{12} = 12.555$$
$$k_{22} = 13.989$$

The mass coefficients are the same as calculated previously. The solution of the polynomial:

$$g(W^2)^2 - q\,W^2 + 1 = 0 \qquad 5.11)$$

now gives the two frequency results as:

$$W_1 = 4.00 \iff p_1 = 3.25 \text{ Hz}$$
$$W_2 = 20.30 \iff p_2 = 16.51 \text{ Hz}.$$

The error in the first bending frequency result has now been reduced to about five percent (compared to the test value of 3.1 Hz), while the error in the second bending frequency result is about 30 percent (compared to 12.7 Hz).

The first bending frequency result, obtained by the simple Rayleigh method was in error by about 16 percent. The use of the two-mode Rayleigh-Ritz procedure has greatly improved the first bending result, especially when the accurate nonlinear stiffness distribution 5.8) is used in the calculation of the stiffness coefficients.

Once the two values of W^2 have been determined, the modeshape approximation can be found. The method of determining the modeshape is typical of many eigenvalue - eigenvector problems.

To determine the first bending modeshape, the value of W^2 corresponding to $W_1 = 4.00$ is subsituted into the matrix equation:

$$\begin{bmatrix} k_{11} - W_1^2 m_{11} & k_{12} - W_1^2 m_{12} \\ k_{12} - W_1^2 m_{12} & k_{22} - W_1^2 m_{22} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad 5.12)$$

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The values of A_1 and A_2 must then be determined for the COR-TEN tower:

$$\begin{bmatrix} 3.648 & -1.957 \\ -1.957 & 0.981 \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Since the determinant of the 2 x 2 matrix is equal to zero, a unique solution for A_1 and A_2 does not exist (Ref. 6). A value for either A_1 or A_2 must first be assumed, and then the other variable solved in terms of the assumed one. In this case A_2 is assumed to equal one, and the equation:

$$3.648\ A_1 \ -1.957\ A_2 = 0$$

is used to solve for A_1 with the result:

$$A_1 = \frac{1.957}{3.648} A_2 = 0.537.$$

The first bending modeshape approximation is then given by:

$$\phi_1(\xi) = 0.537\ \psi_1(\xi) + 1.000\ \psi_2(\xi)$$
$$0 \leq \xi \leq 1.$$

with the two polynomials ψ_1 and ψ_2 given previously on page 61.

Similarly the second bending modeshape can be found by substituting the value of W_2^2 into 5.12) and solving for a second set of coefficients A_1 and A_2. Following this procedure the second bending modeshape is approximated by:

$$\phi_2(\xi) = -0.889\ \psi_1(\xi) + 1.000\ \psi_2(\xi)$$
$$0 < \xi < 1.$$

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Figure 5.3 shows plots of these two modeshapes, compared to the modeshapes obtained from the more accurate MITGUY computer code. This figure shows that the first bending modeshape is quite accurate, but the second shape is relatively inaccurate. The error in the second mode gives an idea of the reason for the second mode frequency inaccuracy, using this method. In order to get more accurate results for the second bending frequency and modeshape one could either try different ψ_1 and ψ_2 functions or add more approximating ψ_i functions to the two that are now used.

In order to use a simple hand method which gives more accurate results than this method, one is encouraged to choose a more accurate ψ_2 function, which better approximates a second bending modeshape.

Another way to get more accurate results is to use more ψ_i functions, i.e., add two more functions to the two which are already used. The four-mode Rayleigh-Ritz computer code will now be used.

Rayleigh-Ritz Computer Code

This computer code uses four degrees of freedom. It is suggested that the analyst use an N degree of freedom method to accurately determine the first N/2 bending modes. With this four degree of freedom method, accurate determination of the first two bending frequencies and modeshapes should be possible.

A listing of the computer code as well as an example data input deck for the COR-TEN is shown in Appendix B. The tower can be divided into N segments, with constant values of mass distribution and stiffness distribution input along each segment. The user must first input the number of segments N. In addition the user inputs the tip mass ratio μ and guy wire properties ($\bar{K}_c$ and ξ_a are zero for the COR-TEN).

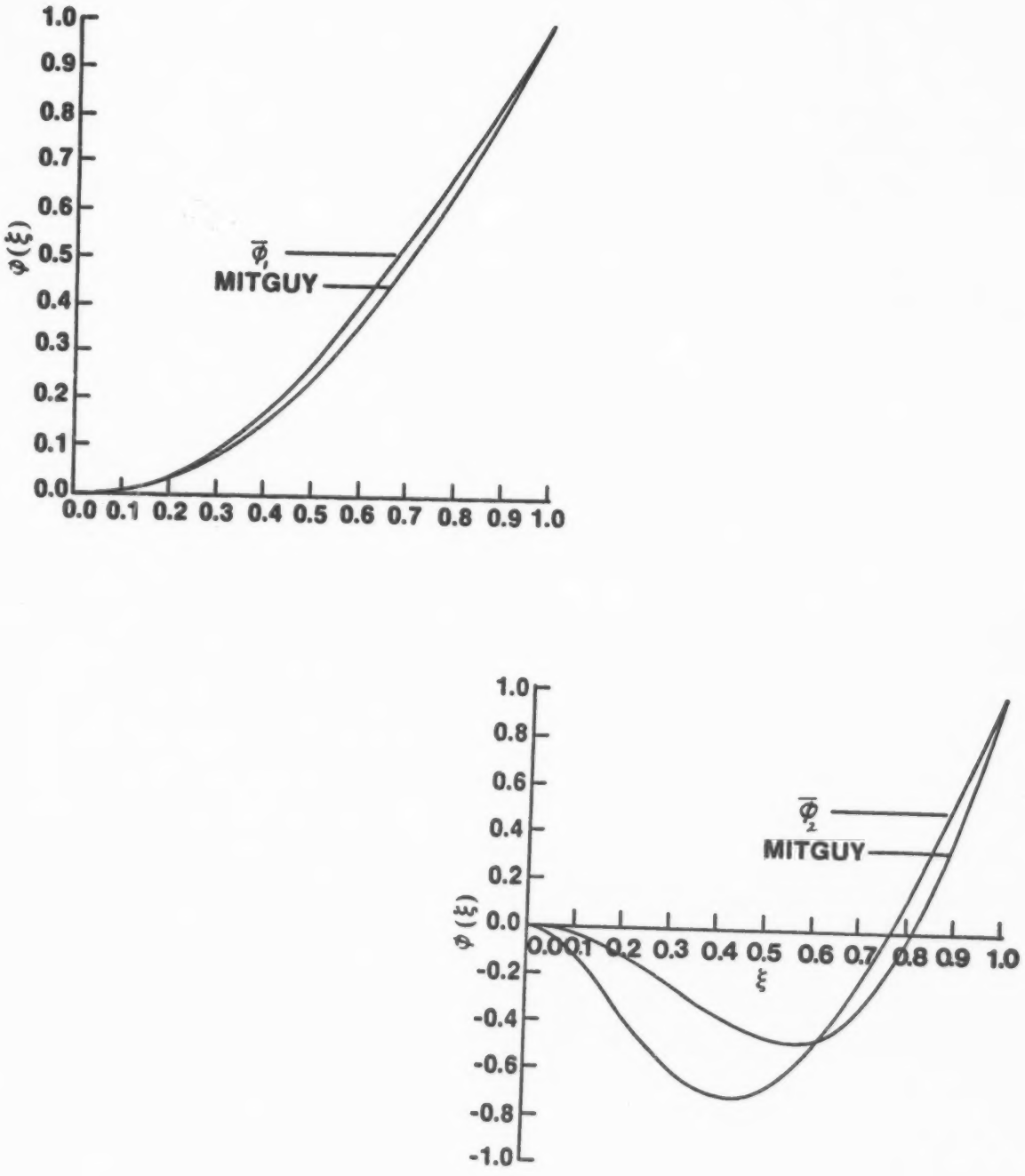

Figure 5.3: Modeshape Approximations From MITGUY and The Simple Rayleigh-Ritz Hand Method (ϕ_i)

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Next the mass and stiffness distributions $h(\xi)$ and $f(\xi)$ are approximated by constant values along each segment. Further details of the data input are also listed in Appendix B.

As with the Rayleigh computer code, several segments must be used in order to assure better accuracy of the stiffness and mass inputs and thus the frequency results. Table V shows the results for five, ten, and twenty tower segments.

For the most accurate results using twenty segments, the second bending frequency estimate is much closer to the test value of 12.7 Hz, than the result given by the simple two-mode method. The use of a higher degree of freedom method causes lower frequency estimates, closer to the exact values, and better estimates for higher modes.

TABLE V: RESULTS FOR THE COR-TEN TOWER USING THE RAYLEIGH-RITZ COMPUTER CODE.

Different Numbers of Segments N are Shown.

N = 5 Segments

p_1 = 3.48 Hz.
p_2 = 14.28 Hz.

N = 10 Segments

p_1 = 3.26 Hz.
p_2 = 12.91 Hz.

N = 20 Segments

p_1 = 3.22 Hz.
p_2 = 12.60 Hz.

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Programs ISCOE and VIBES

This simple code, developed at Oregon State University, has been written for a TI-59 programmable calculator. The program has been divided into two parts: the first part (ISCOE) evaluates the inertia and stiffness coefficients for use in the free vibration equation of Section 2.4. The user must first input the tower properties, soon to be described. The second part (VIBES) solves the free vibration equation for the system natural frequencies.

Once the first program (ISCOE) has been loaded into the calculator, the user begins the input of the tower properties. The first step is the selection of the type of tower base: either a cantilever beam base or a balljoint base (pinned beam). For the COR-TEN tower a cantilever base is chosen.

The user then enters the data for any guy cables, however, for the unguyed COR-TEN tower this step can be skipped. Next, the stiffness and mass properties for the tower alone are input. At this stage the user can either input data for a nonuniform section, in which the mass and stiffness distributions vary, or input data for a uniform section in which the mass and stiffness properties are constant.

For the COR-TEN tower the nonuniform tower option must be selected and the tower divided into N segments, with constant values of mass (ρA) and stiffness (EI) input for each segment. As described in Appendix C, both bending and torsional tower properties are input. The user must determine the torsional stiffness GJ and torsional mass moment of inertia I_m. The input of both bending and torsional properties allows the determination of both bending and torsional natural frequenies, as shown in Section 2.4.

After the tower properties have been input, the rotor and nacelle properties are entered. The user must determine I_{xx}: the mass moment of inertia of the rotor-nacelle about the x-axis; I_{zz}: the moment of

inertia about the z-axis; $M_r + M_n$: the total mass of the rotor plus nacelle; and q: the distance from the tower centerline to the machine c.g. location as shown in Figure 2.5. For the case of the COR-TEN tower, however, only the tower top mass $M_r + M_n$ is input, because the tower was tested with only a top plate, i.e., without a machine on top. The effects of I_{XX}, I_{ZZ}, and q are not considered in this case.

After these steps, the program calculates and prints the mass and stiffness coefficients in the free vibration equation of Section 2.4. The second program is then loaded into the calculator. The mass moment of inertia of the rotor about its spin axis I_r is then entered. For the COR-TEN tower which was tested without a machine on top I_r is set to zero.

Then, the stiffness and inertia coefficients, calculated in the first program, are input into this second program. After these data values have been entered, the program computes and prints the system frequencies for zero and infinite rotor speeds. This code takes into account the effects of rotor speed on system natural frequencies. The user can also obtain system frequencies for various rotor speeds. For the COR-TEN tower the effects of rotor speed cannot be taken into account, since the tower was tested with only a concentrated weight on top. For this case one inputs a value of zero for I_r, and only the frequencies for zero speed are obtained.

More details on the data input and use of the codes are given in Appendices C and D.

For the COR-TEN tower, the properties were input by using N=20 segments. Values for EI, GJ, m, and I_m were input along each segment. Also, a concentrated weight of 50 lb was located at the tower top, so a concentrated mass of 1.55 lb-s^2/ft was input into the program.

After the second code was run the system natural frequencies were output for zero rotor speed. From Figure 2.5 it can be seen that tower motion is allowed (in this model) in the y direction and x direction. The first two tower bending frequencies for either direction are:

$$W_1 = 3.31 \text{ Hz}$$
$$W_2 = 13.15 \text{ Hz}$$

The results are very close to the tower test results; the first bending frequency is in error by about seven percent (compared to the 3.1 Hz test result). The error in the second bending frequency is about four percent (compared to the 12.7 Hz test value). Test results for the tower torsional frequencies were not obtained, so that a direct comparison could not be performed.

The analysis of the COR-TEN tower, using this code, shows that very good results for the first and second bending frequencies can be obtained without use of a large computer program. Also, this TI-59 calculator code takes into account the effects of rotor spin rate, machine c.g. location, etc. for towers with machines. The effects of these parameters could not be shown in this section, since the tower was tested without a machine on top.

The last analytical method of Section 2.0, Program MITGUY, will now be used to analyze the COR-TEN tower. This program uses the Matrix Iteration method, as explained in Section 2.5.

Analysis Using Program MITGUY

A listing of this program, as well as the data input for the COR-TEN tower is shown in Appendix D. As can be seen from the data input, the user must first input the tower height in inches (660.0 in.). Also the user inputs the number of tower segments N; in this case N=20 segments were used.

After these two values have been input, the user inputs values of stiffness, weight distribution, and the assumed mode deflections at that segment for both the first and second bending modes. The values of stiffness and weight distribution have the units lb-in.^{2} and lb/in., respectively.

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The user chooses approximations to the first and second bending mode shapes and inputs these into the program. Each segment of the tower will be given two displacement values, one corresponding to the first mode, and the other corresponding to the second. These two values are input on the same lines as the stiffness and weight distribution values.

After these values have been input, the concentrated top weight is input (the last value in the data input) in pounds.

This method is an iterative procedure, and the speed of convergence depends, in part, on the accuracy of the input modeshape approximations. For the first computer run for the COR-TEN tower, the first two modeshape approximations were simply input as constant deflections with a value of one, as seen in the first data file (Appendix E). The results of this iteration give values for the first two bending frequencies of:

$$p_1 = 3.23 \text{ Hz}$$
$$\text{and } p_2 = 12.48 \text{ Hz};$$

the computations took 1.123 seconds on a Control Data Computer.

The COR-TEN case was rerun using the set of modeshape approximations, shown in Figure 5.3. These modeshape approximations represent the first and second modeshapes more accurately than the approximations in the first data set. The computation time was reduced to 1.023 seconds and the same frequency results were obtained. Even with the more accurate modeshape approximations, the execution time was only reduced slightly. From this analysis it can be concluded that the extra effort involved in obtaining accurate modeshape inputs does not greatly reduce the execution time or computer cost. The analyst is encouraged to input "ones" as in the first data set example. By the iterative approach explained in Section 2.5, accurate frequency and mode shape results will still be obtained when less accurate modeshape approximations are input.

This program can also be used to analyze guyed towers. If a guyed tower is being analyzed, values for the guy wire cross sectional area A' ($in.^2$), elastic modulus E ($lb/in.^2$), length ℓ (in.) and angle θ between tower and guy wire (radians) would be input. These values are placed in the data file after the tower weight and stiffness, as shown in the second data set in Appendix E.

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6.0 EFFECTS OF VARIOUS PARAMETERS ON TOWER NATURAL FREQUENCIES

In this section the effects of changes in various parameters on the tower or system (tower + machine) natural frequencies will be investigated. The effects of such parameters as tower top mass, guy wire stiffness and tension, and guy wire attachment level will be investigated.

Very often it is impossible to get accurate measurements of all of the parameters and properties needed for input into the analysis methods. In such cases the analyst or designer should have a good feel for the errors in the frequency results, due to errors in the data input. The first parameter that will be investigated is the tower top mass.

Effects of Tower Top Mass

In order to study the effects of this parameter the Rayleigh quotient will again be written:

$$p^2(\phi) = \frac{\int_0^L EI(x)[\phi'']^2\,dx}{\int_0^L \rho A(x)[\phi]^2 dx + M\,[\phi(L)]^2} \qquad 6.1)$$

Specific attention should be payed to the term:

$$M[\phi(L)]^2$$

in the denominator of the Rayleigh quotient. From this expression it can be seen that the effect of the top mass M is to add an extra term to the kinetic energy of the system. The result of this extra term is to lower the natural frequencies of the system, because the denominator in the Rayleigh quotient becomes larger. The extent of this change in natural fr quency will depend upon both the magnitude of M and the value of $\phi(L)$: the deflection of the tower top. As will now be shown, changes in top mass have an influence on both the first and second bending frequencies.

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To show the effects of added top mass, a Rohn 45 GSR double guyed tower shown in Figure 6.1 was tested with various top weights. The four-mode Rayleigh-Ritz computer code was also run for this tower with the same tower top weights. Unfortunately the second bending test results for this tower were not obtained.

As can be seen in Table VI, the analysis results agree well with the test results for the first bending frequency (P_1). The first bending frequencies decrease steadily with increasing tower top weight. From Figure 6.2 it can be seen that the second bending frequencies (P_2) also decrease with increases in tower top weight.

TABLE VI: COMPARISON OF TEST RESULTS AND ANALYTICAL RESULTS (FROM THE RAYLEIGH RITZ-COMPUTER CODE) FOR THE FIRST BENDING FREQUENCY OF A ROHN 45 GSR DOUBLE GUYED TOWER. ALSO SHOWN ARE SECOND MODE RESULTS FROM THE COMPUTER CODE.

	Test Results P_1	Analytical Results P_1	Analytical Results P_2
0 lb	——	9.1 Hz	20.3 Hz
100 lb	——	6.2 Hz	18.5 Hz
200 lb	5.0 Hz	5.2 Hz	18.2 Hz
300 lb	4.5 Hz	4.5 Hz	18.0 Hz
400 lb	4.0 Hz	4.1 Hz	17.9 Hz
500 lb	3.7 Hz	3.8 Hz	17.8 Hz
600 lb	3.5 Hz	3.5 Hz	17.8 Hz
700 lb	3.2 Hz	3.3 Hz	17.7 Hz

As can be seen in Figure 6.2 the frequencies decrease less and less with increasing weight. The greatest decrease in frequency occurs for the first few hundred pounds of weight. The good correlation between test results and analytical results shows that the Rayleigh-Ritz four-mode computer code treats the top mass effects accurately.

A question remains as to the magnitude of errors in frequency results due to a given error in tower top weight. For this example case, suppose that the estimated tower top weight is 500 lb plus or minus 50 lb (a 10% error).

$$\text{WEIGHT} = 500 \text{ lb} \pm 50 \text{ lb}$$

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Figure 6.1: Rohn 45GSR Double Guyed Tower

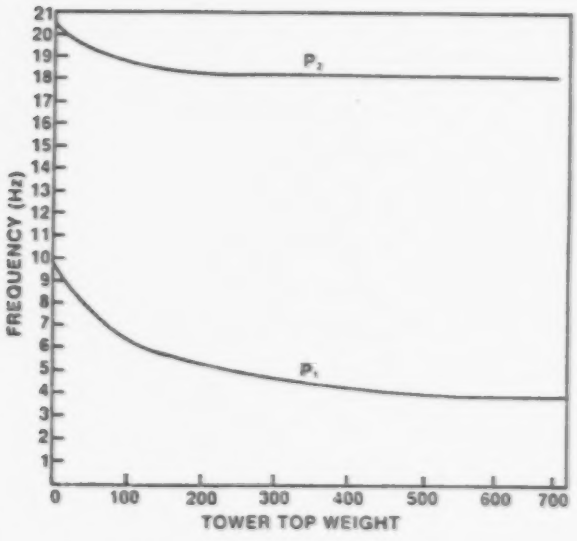

Figure 6.2: Tower Top Weight Effects for a Rohn 45 GSR Double Guyed Tower.

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From Table VI, it can be seen that the computed first bending frequency for M = 500 lb is about 3.7 Hz, while the frequency for M = 550 lb is about 3.6 Hz. An error in the estimated tower top weight of 10 percent causes a maximum error of about 3 percent for the first bending frequency.

In summary, accurate measurements of tower top weight are not always possible. In such cases, the designer should try to estimate the errors in weight measurements and then try to obtain corresponding errors in the frequency estimates. The designer should try to determine a range of possible tower frequencies by knowing the largest and smallest tower top weights possible.

Effects of Guy Wires

The effects of the guy wires were to add an extra term to the potential energy of the system. The Rayleigh quotient will again be written; however, now the guy wire effects will be included:

$$p^2(\phi) = \frac{\int_0^L EI(x)[\phi'']^2\,dx + K_c\,[\phi(a)]^2}{\int_0^L \rho A(x)[\phi]^2\,dx + M\,[\phi(L)]^2} \qquad 6.2)$$

Specific attention should be paid to the term:

$$K_c\,[\phi(a)]^2$$

in the numerator of the Rayleigh quotient. From this expression it can be seen that the effects of the guy wires are to add an extra term to the potential energy, which causes the numerator to become larger. The tower natural frequencies are thus increased. The guy wires are assumed to be massless stiffening members. They do not add mass to the system and so no additional terms appear in the denominator of Equation 6.1).

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Recall that for a set of three guy wires attached at the same level and spaced 120° apart:

$$K_c = \frac{3}{2}\left(\frac{A' E'}{\ell}\right) \cos^2\theta \qquad 6.3)$$

as shown in Figure 2.4. The guy wire stiffening effect is seen to be a function of A': the guy wire cross sectional area, E': the guy wire elastic modulus, ℓ: the guy wire length, θ: the angle the guy cable makes with the ground and $\phi(a)$: the tower deflection at the guy attachment point.

As stated in Section 3.4, accurate measurements of A' and E' are not always possible, especially for wire rope which consists of strands. Also, changes in the guy attachment point-a can change the stiffening effect of a given set of guy wires.

In order to show the effects of guy wire parameters on tower natural frequencies the Rayleigh-Ritz computer code was run for several values of dimensionless guy coeffecient $\bar{K}_c$ and ξ_a, for an imaginary tower. The tower has a uniform dimensionless mass and stiffness distribution and length of 1. Figure 6.3 shows the results of these computer runs for one set of guy wires and values of $\bar{K}_c$ equal to 50, 100, 150, 200, and 300. Also, values of ξ_a from 0.4 to 1.9 are shown. The value of top mass ratio μ was set at 0.5.

Recall that

$$\bar{K}_c = \frac{K_c L^3}{EI_0},$$

where K_c is defined in Equation 6.3), L is the tower height and EI_0 the tower base stiffness value.

Also recall that

$$\xi_a = a/L,$$

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where a is the guy cable attachment height.

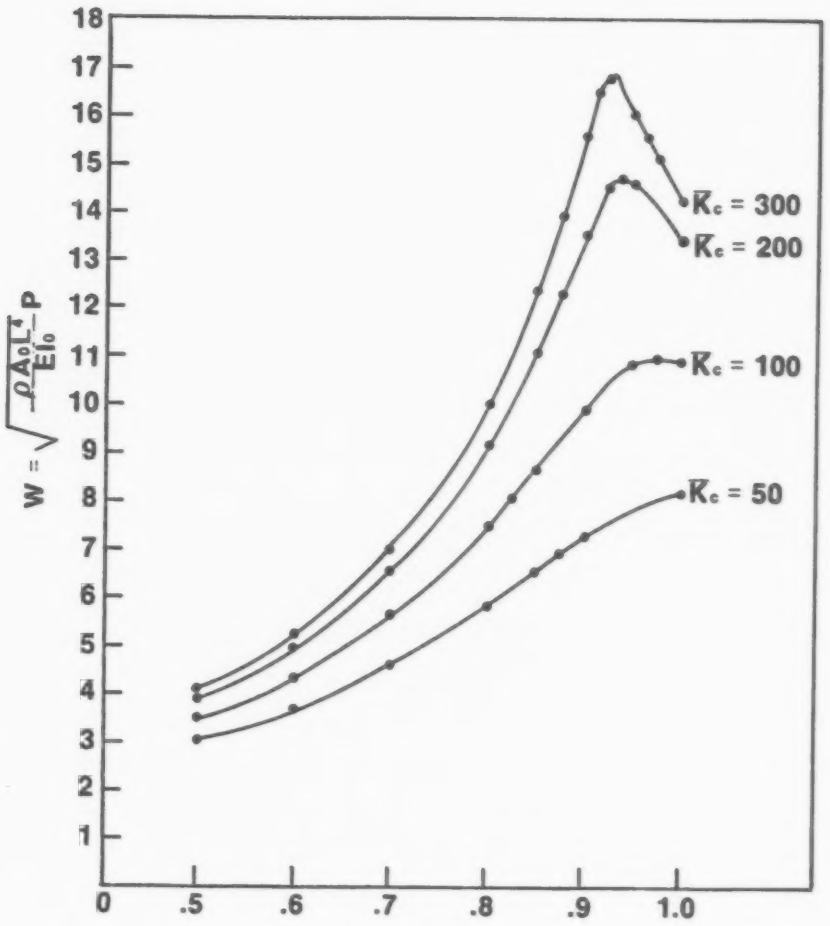

Figure 6.3: Effects of Guy Wire Parameters on the Tower Fundamental Frequency for an Imaginary Tower using the Rayleigh-Ritz Computer Code.

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Figure 6.3 shows the effects of increasing guy stiffness coefficient parameter $\bar{K}_c$ and attachment level ξ_a on the tower first bending frequency. The frequencies tend to increase as the stiffness coefficient parameter $\bar{K}_c$ is increased. For the higher values of $\bar{K}_c$ the frequencies increase with increasing ξ_a up to a certain point and then either level off or decrease. There appears to be an optimum guy attachment level ($\xi_a \approx 0.9$) for the best stiffening effect. Note should be made of the fact that the values for frequency shown in this figure are dimensionless quantities. The true frequency values for a particular tower with uniform mass and stiffness distributions would be obtained by multiplying the W values by:

$$\sqrt{\frac{EI_0}{\rho A_0 L^4}}$$

for that particular tower. Here EI_0 is the value of tower stiffness distribution at the base, and ρA_0 is the value of tower base mass distribution.

Emphasis must be placed on the fact that these guy wire effects are only valid if the guy wires are tensioned sufficiently so that guy wire tower interaction will not occur. When the fundamental bending frequency of a guy wire is close to the tower fundamental frequency, significant coupling can occur, which changes the stiffening effects of the guys. In such cases, expression 6.3) for the guy wire stiffness coefficient will no longer apply. This coupling effect will now be addressed.

Guy Wire-Tower Interaction

In general, the guy wire stiffness coefficient K_c can be shown to depend on the pretension of the guy wires, and can be expressed (Ref. 5):

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$$K_c = \frac{3}{2}\left(\frac{A' E'\alpha}{\tan(\alpha\ell)} \cos^2\theta + \frac{T_0 \lambda}{\tan(\lambda\ell)} \sin^2\theta\right) \qquad 6.4)$$

for three guys spaced 120° apart.* Here, T_0 is the pretension force in the guys. A' and E' are the cross sectional area and elastic modulus of the guys, respectively, and θ is the angle the guy wires make with the ground. Here, an assumption is made that A', E', Θ, and T_0 are the same for each guy wire.

The special parameters α and λ are defined as:

$$\alpha = \sqrt{\frac{m' p^2}{A' E'}} \qquad 6.5)$$

$$\lambda = \sqrt{\frac{m' p^2}{T_0}} \qquad 6.6)$$

where m' is the guy cable mass distribution and p is the frequency of tower motion.

The fundamental frequency of a guy wire can be shown to be (Ref. 5):

$$W = \frac{\pi}{\ell}\sqrt{\frac{T_0}{m'}} \qquad 6.7)$$

When the guy fundamental frequency is well separated from the tower frequency, it can be shown that expression 6.4) reduces to the familiar form:

$$K_c = \frac{3}{2}\left(\frac{A' E'}{\ell}\right)\cos^2\theta \qquad 6.8)$$

---

* For four guys spaced 90° apart the factor 3/2 in 6.4) becomes a 2.

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The effects of guy wire tension T_0, or guy wire fundamental frequency W, on the stiffness coefficient K_c can be shown. The parameter λ can be rewritten by combining Equations 6.6) and 6.7) so that:

$$\lambda \ell = \frac{\pi p}{W} \qquad 6.9)$$

From this expression it can be seen that as the guy pretension T_0 is adjusted so as to cause the guy wire frequency W to approach the tower frequency p, the quantity $\lambda \ell$ in equation 6.9) approaches π. The quantity tan ($\lambda \ell$) becomes smaller and smaller, causing the second term in 6.4) to become large. These effects will now be illustrated for the COR-TEN guyed tower.

Guy Wire Effects for the COR-TEN Tower

From Section 3.3 it was mentioned that the guy wires are solid steel rods with a cross sectional area A' of 0.44 $in.^2$ and E' = 30 x 10^6 $lb/in.^2$. The angle that the guy cables make with the ground is 45° and the length ℓ is 679 in. From these values, and using expression 6.8) the guy wire stiffness coefficient K_c was calculated to be 233,018 lb/ft.

As can be seen from expression 6.4) however, the stiffness coefficient actually changes with guy pretension T_0. Table VII shows changes in this coefficient with various assumed pretension levels, with the assumption that the tower frequency p remains constant at 2.4 Hz.* The values in this table are not actual test results but are calculated from Equation 6.4).

Table VII shows that when the guy wire fundamental frequency approaches the tower fundamental frequency (2.4 Hz), the guy wire stiffness coefficient

* This assumption may be poor because the solution of this coupled guy wire tower problem requires solution of the tower equation of motion in conjunction with the equation for the guy wire effects. The tower frequency will thus not be a constant.

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TABLE VII: VARIATION OF GUY WIRE STIFFNESS COEFFICIENT K_c WITH GUY WIRE PRETENSION T_0. THE GUY WIRE FUNDAMENTAL FREQUENCY W IS ALSO SHOWN.

K_c (lb/ft)	T_0 (lb)	W (Hz)
196,300	3470	2.401
199,600	3480	2.405
214,300	3490	2.408
220,000	3500	2.412
232,000	4000	2.578
232,600	5000	2.882

K_c differs from the value given by 6.8) by increasing amounts. The use of this simple expression is no longer valid. The resulting tower fundamental frequency will be different from the value given by using the simple methods and expression 6.8).

The actual solution of the coupled tower guy wire problem is beyond the scope of this report. The designer or analyst should adjust the guy wire pretension so that the guy wire fundamental frequency will be well separated from the tower fundamental frequency. Only when this criterion is satisfied can the simple methods be used, with expression 6.8) for the guy wire stiffness coefficient. If any significant coupling between guy wires and the tower occurs, a more complex analysis is needed.

More information on guy wire-tower interaction is given in Reference 1. In this report, the authors recommend that the guy wire frequency be separated from the tower frequency by at least 30 percent. In Reference 1 actual test results were compared to analytical results for a certain guyed tower. The tower fundamental frequencies were obtained for various guy tension levels.

Now that the effects of some of the tower parameters have been investigated, actual test results will be compared to analytical results for four towers. For the last tower analyzed, it will be shown that the simple analysis methods give inaccurate results compared to test results. A possible reason for these inaccuracies is guy wire-tower interaction just described.

7.0 COMPARISONS OF ANALYTICAL RESULTS TO TEST RESULTS

In this section results from the analysis methods of Section 2.0 will be compared to test results, for the towers previously presented. For the case in which the simple methods give poor results for the Rohn SSV tower, the complex finite element code SAPIV will be used to analyze the tower.

SAPIV is a complex and large computer code which models every member of truss type towers. As will be shown, it can be used for such towers as the Rohn SSV, in which the effects of the cross members are hard to determine, for use in the simple methods.

The emphasis of this report is in the identification and evaluation of various simple analysis methods (compared to SAPIV). In this section these simple methods will be evaluated.

COR-TEN Freestanding Tower Results

The results from each of the methods for this tower are also presented in Section 5.0, where the correct use of each method is presented. Table VIII shows a summary of results for this tower. The first bending frequency is p_1 and the second is p_2.

TABLE VIII: .COR-TEN FREESTANDING TOWER RESULTS SUMMARY

Method	p_1 (Hz)	Error	p_2 (Hz)	Error
Test Results	3.1		12.7	
Simple Rayleigh Hand Method	3.6	16%		
Rayleigh Computer Code	3.7	19%		
Simple Rayleigh-Ritz Hand Method	3.3	6%	16.5	30%
Rayleigh-Ritz Computer Code	3.2	3%	12.6	1%
Programs ISCOE and VIBES	3.2	3%	13.2	4%
Program MITGUY	3.2	3%	12.5	2%

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From Table VIII it can be seen that the Rayleigh method gives a very rough estimate of the first bending frequency of the tower. This method could be used to find the second bending frequency if a good second modeshape approximation could be used in the Rayleigh quotient (see Section 2.1).

As was mentioned in Section 2.2, the simple Rayleigh method, using the assumed modeshape given by Equation 2.5), Section 2.2, gives poor results for towers in which the stiffness distributions taper severely. For beams which have constant mass and stiffness distributions, this method gives much better results, as can be seen in Table I, Section 2.0, where a simple cantilever beam was analyzed.

The Rayleigh-Ritz hand method gives much more accurate results than the Rayleigh method, for the first bending frequency. For the second bending frequency, however, the results are very inaccurate, as seen by the 30% error. As was mentioned previously, the designer is urged to use an N degree of freedom method to accurately determine the first N/2 bending frequencies. The Rayleigh-Ritz hand method is a two degree of freedom method.

The Rayleigh-Ritz computer code, a four degree of freedom method (see Section 2.3) gives very good results for both the first and second bending frequencies. This program, though, is written for a large computer (such as a Control Data Corporation 7600 Computer).

Program ISCOE and VIBES give very good results for both the first and second bending frequencies. These programs are designed to be used on a Texas Instruments (TI-59) programmable calculator. The error for the second bending frequency is somewhat higher than results from the Rayleigh-Ritz computer code or Program MITGUY. These codes do give excellent results for the first bending frequency, and can be used on a hand-held programmable calculator.

Finally, Program MITGUY, a matrix iteration procedure, is also seen to give very good results for this COR-TEN pole tower.

These results show that the simple methods can be used to good advantage to analyze pole towers. The first two methods give large errors because the tower has a large taper in stiffness distribution. For towers with constant stiffness values (uniform distributions) even these simpler methods would give good results (see Table I, Section 2.2).

Now that the results for a freestanding pole tower have been presented, the results for a truss tower will be shown.

Rohn SSV Freestanding Tower Results

Table IX shows the results for the Rohn SSV tower. The analysis of this tower with the hand methods (simple Rayleigh hand method and simple Rayleigh-Ritz hand method) is difficult, because of the highly tapered stiffness distribution.

As can be seen from the table, the Rayleigh-Ritz computer code can be used to find better values for the first bending frequency. The tower is divided into 20 segments and constant values of mass and stiffness input along each segment.

TABLE IX: ROHN SSV FREESTANDING TOWER RESULTS SUMMARY

Method	p_1	% Error	p_2	% Error
Test Results	3.3		12.7	
Simple Rayleigh Hand Method	3.8	19	—	—
Rayleigh Computer Code	4.0	21		
Simple Rayleigh-Ritz Hand Method	3.6	12	22.3	76
Rayleigh-Ritz Computer Code	3.4	6	15.5	22
Programs ISCOE and VIBES	3.5	9	16.3	28
Program MITGUY	3.4	6	14.8	17
Program SAPIV	3.2	3	13.0	2

Programs ISCOE and VIBES give just fair results for the first bending frequency, and poor results for the second. The results for the first and second bending frequencies are less accurate from this code, than the results from MITGUY or the Rayleigh-Ritz computer code.

As can be seen from this table, all of the codes and methods (except SAPIV) give less accurate results for this tower than for the COR-TEN freestanding tower.

With these simple methods, this tower was modeled as a beam, neglecting the effects of the cross members in the stiffness distribution. The cross section moment of inertia was calculated, by taking into account the three legs only, as described in Section 3.2. The actual stiffness distribution of this tower is more complex than this beam approximation because of the cross memebers. For these reasons, the complex finite element code SAPIV was used to model this tower, taking into account the effects of every member. The improvement in the first and second bending frequency is very evident.

Now that the results for two types of freestanding towers have been shown, the results for two guyed towers will be described.

Rohn 25G Results

Table X shows the results for the Rohn 25G tower. The simple Rayleigh-Ritz hand method gives good results for the first bending frequency but fair results for the second frequency. The utility of this method is that it can be used very quickly to get a good estimate of the first bending frequency of guyed or unguyed towers.

TABLE X: ROHN 25 SINGLE GUYED TOWER RESULTS SUMMARY

Method	p_1	Error	p_2	Error
Test Results	2.6 Hz		7.8	
Simple Rayleigh-Ritz Hand Method	2.8 Hz	8%	9.0	15%
Rayleigh-Ritz Computer Code	2.6 Hz	0%	8.8	13%
Programs ISCOE and VIBES	2.6 Hz	0%	8.6	10%
Program MITGUY	2.6 Hz	0%	8.6	10%

The Rayleigh-Ritz computer code, which uses four degrees of freedom, gives excellent results for the first mode and fair results for the second mode. Program MITGUY, the matrix iteration procedure, gives about the same results as the other two codes.

The ten percent inaccuracy in the second mode results is thought to be caused by inaccuracies in the measurement of tower parameters. Also, neglect of the effects of the cross members in this tower could affect the frequency results, similar to the Rohn SSV.

Emphasis must be placed in the fact that the guy wire fundamental frequencies are well separated from the tower fundamental frequency. Because there is very little tower-guy wire coupling, the effects of the guys are straight forward: an extra term is added to the potential energy (stiffness) of the tower (see Section 2.3). For cases in which guy wire-tower coupling occurs, these simple methods would have to be modified to take this effect into account. A complicated coupling problem must be solved, which is beyond the scope of this report.

The next tower results summary will show the possible effects of guy wire-tower interaction.

COR-TEN Guyed Tower Results

As was mentioned in Section 4.0, the guy wire fundamental frequencies are close to the tower fundamental frequency for this case. One would expect the analysis methods to give worse results for this case than the case of the Rohn 25G tower.

Table XI show the results for this tower from the analysis methods, compared to the test result for the tower fundamental frequency of 2.4 Hz. In each of these methods, the guy wires have been treated very simply as stiffening members which add an extra term to the potential energy of the system (see Section 6.0).

As shown in Section 6.0, the guy wire fundamental frequencies change with guy pretension level. These guy wires were each pretensioned to approximately 4000 lb, so that each had a fundamental frequency of about 2.6 Hz.

TABLE XI: COR-TEN SINGLE GUYED TOWER RESULTS SUMMARY

Method	p_1 (Hz)	Error
Test Results	2.4	—
Simple Rayleigh-Ritz Hand Method	3.2	33%
Rayleigh-Ritz Computer Code	2.7	13%
Programs ISCOE and VIBES	2.9	21%
Program MITGUY	2.7	13%

In Reference 1 it is mentioned that the guy wire fundamental frequency should be separated from the tower fundamental frequency by at least 30 percent. For this case, the guy frequency is only separated from the tower frequency by about 8 percent. The possibility of guy wire tower interaction is very great and may explain why these methods give less accurate results than for the Rohn 25G tower.

8.0 SUMMARY OF THE ANALYTICAL METHODS

In the last section, it was pointed out that the simple analysis methods work well for pole towers. For towers with many cross members (truss-type towers) these methods give less accurate results because simple stiffness distributions EI(x) are hard to determine. A large finite element program such as SAPIV, which models every member, gives much more accurate results for such towers, especially for the second bending frequency. The methods MITGUY, ISCOE and VIBES, and The Rayleigh-Ritz computer code do give fair results for the first bending frequency of truss type towers (see Table IX).

A summary of the usefulness, and accuracy of each method will now be given.

Simple Rayleigh Hand Method

This method is only useful for a rough estimate of the first bending frequency of pole type towers. Even for pole type towers it gives poor results when the stiffness distribution tapers, as in the freestanding COR-TEN tower. This method gives much more accurate results for pole towers with uniform mass and stiffness distributions. This method is only useful for towers in which simple functions can be used to estimate the stiffness and mass distributions EI(x) and $\rho A(x)$, respectively, such as linear mass and stiffness distributions.

Rayleigh Computer Code

This method gives about the same results as the last method. However, this program can be used for towers having more complicated stiffness and mass distributions, in which simple functions are hard to find to estimate EI(x) and $\rho A(x)$. The tower can be divided into N segments, and constant values for EI and ρA input along each segment.

The disadvantage of this method is that for highly tapered or highly nonlinear mass and stiffness distributions, many segments must be used. Also, a single modeshape assumption is used to approximate the exact bending modeshape. The accuracy of the resulting frequency depends, to a large extent, on how accurately this assumed shape approximates the exact first bending modeshape.

Simple Rayleigh-Ritz Hand Method

This hand method uses a linear combination of two approximating shape functions to estimate the modeshape. Because the method utilizes two degrees of freedom, it gives more accurate results than the simple Rayleigh hand method. This method gives good results for the first bending frequency of pole towers with constant or tapered stiffness distributions. The second bending frequency results for tapered towers are poor, however. This method gives better second frequency results for towers with constant values of mass and stiffness along the tower (see Table I). This method can also be used for guyed towers.

Rayleigh-Ritz Computer Code

This code gives good results for both the first and second bending frequencies for towers having constant or tapered stiffness distributions. The method utilizes four approximating functions (four degrees of freedom). With the use of this number of shape functions, the inaccuracies in the second mode results for tapered towers are reduced, compared to the last method. If the stiffness distribution of the tower is very nonlinear or hard to determine, such as for the Rohn SSV which has many cross members, this method will give inaccurate results, especially for the second bending frequency. A finite element code such as SAPIV is needed.

All of the methods so far have treated the machine at the tower top as a lumped mass.

Programs ISCOE and VIBES

These codes are written for a TI-59 hand-held calculator. These codes give good results for the first bending frequency of most towers. For some towers, the results for the second bending frequencies are poor, such as towers with many cross members.

The great utility of these codes is that accurate first bending results can be obtained for guyed or unguyed towers. Also, the machine c.g. location and rotor spin rate can be entered into the program to show the effects of these parameters on system natural frequencies. Also, the nacelle-rotor mass moment of inertia effects can be accounted for. This method provides more sophistication for determining rotor-nacelle effects on system frequencies. However, the accuracies of modeling these other effects are not yet known, because test results were not obtained to check these effects.

As in the other methods, these codes give less accurate results for towers with complicated mass or stiffness distributions.

Program MITGUY

This program has the advantage of using an iteration technique to find frequencies and modeshapes. As was seen in Section 5.0, two assumed modeshapes are entered into the program, one for the first mode and one for the second. The great utility of this method is that very rough estimates of these shapes can be entered into the program and, by iteration, very accurate estimates of the first and second bending frequencies and mode-shapes can be obtained.

This program can be used to find first and second bending frequencies of guyed or unguyed towers. The machine is treated as a concentrated mass at the tower top. The effects of machine c.g. location, rotor-nacelle mass moment of inertia effects, and rotor spin rate are not taken into account with this method though, as in the last method.

All of these methods (except for the simple Rayleigh methods, which do not handle guyed towers) have treated the guy wire effects in a simple way (see Section 6.0). Problems arise if the guy wires are not tensioned sufficiently to prevent guy wire-tower interaction. These methods give good results for the bending frequencies if the guy wires are sufficiently pretensioned.

9.0 CONCLUSIONS AND RECOMMENDATIONS

As has been seen in the last two sections most of the methods presented in this report can be used to get good estimates of the fundamental frequencies for certain types of guyed and freestanding towers. The methods give especially good results for pole type towers, such as the COR-TEN freestanding tower. Also, the methods give good results for guyed towers such as the Rohn 25G, when the guy wire fundamental frequency is well separated from the tower fundamental frequency, as shown in Table XII.

The more accurate methods of this report (Rayleigh-Ritz computer code, ISCOE and VIBES, and MITGUY) give good results for the second bending frequencies for certain freestanding and guyed towers. The simpler methods of this report (Rayleigh hand method, Rayleigh-computer code) give very rough estimates only for the first bending frequency, as seen in Table XII.

Problems arise in the use of the methods of this report in the analysis of truss towers such as the Rohn SSV. The simplest methods such as the Rayleigh methods and Rayleigh-Ritz hand method give poor results for the first bending frequency of this tower. The more complex methods of this report, such as the Rayleigh-Ritz computer code, Programs ISCOE and VIBES, and MITGUY give good results for the first bending frequency but poor results for the second bending frequency of this tower.

From the results for various towers given in Section 7.0, the Rayleigh-Ritz computer code, ISCOE and VIBES, and MITGUY give the best results among the various methods presented in this report. The Rayleigh-Ritz computer code has been written for a large computer such as an IBM or CDC (Control Data Corporation). Programs ISCOE and VIBES have been written for Texas Instruments (TI-59) hand-held calculators. These programs could be rewritten for other types of calculators. Program MITGUY has also been written for a large computer but is smaller than the Rayleigh-Ritz computer code and more easily programmed.

TABLE XII: SUMMARY OF ERRORS FOR THE FIRST AND SECOND BENDING FREQUENCY RESULTS FOR EACH METHOD COMPARED TO TEST RESULTS

Methods	COR-TEN Freestanding Tower		Rohn SSV Freestanding Tower		Rohn 25 G Guyed Tower	
	p_1:	p_2:	p_1:	p_2:	p_1:	p_2:
Simple Rayleigh Hand Method	16%	—	19%	—	—	—
Rayleigh Computer Code	19%	—	21%	—	—	—
Simple Rayleigh-Ritz Hand Method	6%	30%	12%	76%	8%	16%
Rayleigh-Ritz Computer Code	3%	1%	6%	22%	0%	13%
Programs ISCOE and VIBES	3%	4%	9%	28%	0%	10%
Program MITGUY	3%	2%	6%	17%	0%	10%

The hand methods can only be used to get rough estimates of the first bending frequency for preliminary design considerations. The Rayleigh-Ritz hand analysis is easy to use and gives the best results for the first bending frequency.

More work needs to be performed in order to determine better methods for estimating frequencies of truss towers such as the Rohn SSV. Better methods need to be found to estimate the stiffness distributions of these truss towers, by taking into account each cross member.

Also, easy methods of analyzing guyed towers, when guy wire tower interaction occurs, need to be developed. The possibility of developing a simple coupled analysis needs to be considered. Some work in this area has already been performed and is described in References 1 and 5.

This report has been written to give members of industry an idea of some simpler tower dynamic analysis methods; their correct use and accuracy. Hopefully, an idea of the types of towers for which these methods give adequate results has also been described. The use of huge and expensive finite element computer codes is not always necessary for obtaining good estimates of tower vibration frequencies and modeshapes.

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Appendix A: The Rayleigh Computer Code

This program uses the Rayleigh method to analyze beam type structures. In order to use the program the user must first input an assumed modeshape and the tower mass and stiffness properties. The program reads one of four standard cantilever beam modeshapes from file "MODES" and the tower mass and stiffness properties from file "TWPROP." If the user does not wish to use one of the four modeshapes from file "MODES" he can input a nonstandard modeshape directly from the terminal.

A listing of Program Rayleigh is shown below. During execution of the program the user must answer the question: "Is a nonstandard modeshape to be used?" If the answer to this question is "no," a standard modeshape is read from file "MODES." If the answer is "yes," a nonstandard modeshape is input, by the user from the terminal.

A listing of file "MODES" is shown after the listing of Program Rayleigh. As can be seen, the tower (or beam) is divided into six segments and four values are input at each of the seven nodal positions. The tower location x, modal displacement Y, modal slope dY/dx, and curvature D^2Y/DX^2 must be input.

After the modeshape information has been input, the tower physical properties must be read from the file "TWPROP." An example of this file is shown for the Rohn SSV tower, after the listing of file "MODES." As can be seen, the position (in inches), weight distribution (lb/in.) and stiffness (lb-in.^{2}) are input at each node point.

The tower has been broken up into a series of six equal elements and the stiffness and mass properties evaluated at each of the seven nodes.

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The stiffness values EI_i are input at each node $x_i=iL/N$, where $i = 1,2,\ldots,N$. The tower is then approximated by a series of constant stiffness steps, where each constant stiffness step $EI_{i,i-1}$ is related to the node stiffness EI_i by:

$$EI_{i,i-1} = \frac{EI_i + EI_{i-1}}{2}$$

for values of $i=1,2,3,\ldots,N$. This is performed in the program. The user simply inputs the stiffness at the nodes. Values for weight distribution are input at each node point and are used to get values for lumped mass.

After these values have been input, the concentrated top weight is input. The program then calculates the fundamental tower frequency.

Emphasis should be placed on the fact that the tower can only be divided into six segments, as the program is now written. For towers with highly nonlinear mass or stiffness distributions, more than six segments should be used. The program should be modified to read N nodal positions, where N is any integer. The use of more segments (node points) will increase the accuracy of the frequency results.

The selected modeshape will also increase the accuracy of the frequency results. The user should try each of the four modeshapes from file "MODES" to see which one results in the lowest frequency. If the user is not satisfied with the results from one of these modeshapes, he should input a nonstandard modeshape that he has previously determined.

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```
00100 PROGRAM RAYLGH(INPUT,OUTPUT,MODES,TWPROP,TAPE5=INPUT,TAPE6=
00110+OUTPUT,TAPE7=MODES,TAPE8=TWPROP)
00120 DIMENSION PX(10),G(10),MASS(20),EI(20),DY(10),DDY(10),X(10),Y(10)
00125 REAL MASS,MASSUM,FREQ,MASSN,INB
00130C THIS PROGRAM USES A RAYLIEGH METHOD TO SOLVE FOR FUNDAMENTAL
00140C FREQUENCIES OF BEAM TYPE STRUCTURES. IT READS ONE OF FOUR
00150C ASSUMED MODE SHAPES FROM FILE MODES AND READS THE BEAM
00160C MASS AND STIFFNESS PROPERTIES FROM FILE TOWPROP. THE MODES
00170C SHAPES AND ANALYSIS PROCEDURE IS TAKEN FROM
00180C
00190C
00200C            VOLUME 1
00210C       WIND ENEGY CONVERSION
00220C         DESIGN MANUAL
00230C        ASRL-TR-184-7
00240C      (PREPARED BY MIT)
00250C
00260C
00265 DATA(G(N),N=1,7)/.5,1.,1.,1.,1.,1.,.5/
00270 WRITE(6,10)
00280 10 FORMAT("IS A NONSTANDARD MODE SHAPE TO BE USED(1=YES,0=NO)")
00300 READ(5,*) A
00310 IF(A.EQ.1)20,40
00320 20 WRITE(6,30)
00330 30 FORMAT(/"INPUT SEVEN TOWER POSITION ODAL DISPLACEMENTS",
00340+",FRACTIONAL TOWER LOCATION,MODAL DISPLACEMENT(Y),DY/DX,"
00350+,"DDY/DDX"/)
00360 READ(5,*)(PX(N),Y(N),DY(N),DDY(N),N=1,7)
00370 GO TO 70
00380 40 WRITE(6,50)
00390 50  FORMAT(/"WHICH STANDARD APPROXIMATE FUNDAMENTAL"/,
00400+"MODE SHAPE SHALL BE USED(A=10,B=20,C=30,D=40)\")
00410 READ(5,*) MODSHAP
00420 DO 55 N=1,MODSHAP
00425 55 READ(7,150)DUMBY
00430 READ(7,60)(PX(N),Y(N),DY(N),DDY(N),N=1,7)
00440 60 FORMAT(4F10.2)
00450 70 CONTINUE
00460C
00470C
00480C   THIS SECTION READS AND INTEGRATES THE MASS AND STIFFNESS
00490C  ENERGY TERMS
00500C
00510C
00520 READ(8,80)
00530 80 FORMAT(1H ,"TOWER NAME")
00540 READ(8,90)(X(N),MASS(N),EI(N),N=1,7)
00550 90 FORMAT(2F10.2,E8.3)
00555 READ(8,95) MASSN,INB
00556 95 FORMAT(F10.3,E10.3)
```


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```
00560 WRITE(6,80)
00570 WRITE(6,100)
00580 100 FORMAT(/" HEIGHT  MODE SHAPE(Y)   DY      DDY    MASS     EI ")
00590 WRITE(6,110)(X(N),Y(N),DY(N),DDY(N),MASS(N),EI(N),N=1,7)
00600 110 FORMAT(F6.1,F10.3,F13.3,F8.3,F7.2,E10.3)
00605 WRITE(6,120) MASSN
00606 120 FORMAT(/"NACELL WEIGHT IS",F10.2,"LBS")
00607 WRITE(6,125)INB
00608 125 FORMAT(/"NACELL AND ROTOR MOMENT OF ",/"
00609+INERTIA IS",E10.2,"LB-IN2")
00610 DO 130 ,N=1,7
00620 EISUM=EISUM+G(N)*EI(N)*(DDY(N))**2.
00630 130 MASSUM=MASSUM+G(N)*MASS(N)*(Y(N))**2.
00640C
00650C
00660C  THIS SECTION CALCULATES THE FREQUENCY
00670C
00675 MASSN=MASSN+INB*(DY(7)**2)/((X(7)**2)*(Y(7)**2))
00680 EISUM=EISUM/((X(7)**3)*6.)
00690 MASSUM=(MASSUM*X(7)/(6.)+MASSN*(Y(7)**2))/(384.)
00700 FREQ=SQRT(EISUM/MASSUM)/(6.2832)
00710 WRITE(6,140)FREQ
00720 140 FORMAT(//"THE FUNDAMENTAL BENDING FREQUENCY IS",F5.2/)
00725 150 FORMAT(A10)
00730 END
```

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THESE ARE ASSUMED MODE SHAPES FOR
CANTELEVERED BEAMS

X	Y	DY/DX	DDY/DDX
0.000	0.000	0.000	3.520
0.167	0.048	0.514	2.710
0.333	0.167	0.899	1.930
0.500	0.340	1.160	1.190
0.667	0.551	1.310	0.589
0.833	0.771	1.370	0.169
1.000	1.000	1.380	0.000
0.000	0.000	0.000	2.0
0.167	0.028	0.333	2.0
0.333	0.111	0.667	2.0
0.500	0.250	1.000	2.0
0.667	0.444	1.333	2.0
0.833	0.694	1.667	2.0
1.000	1.000	2.000	2.0
0.000	0.000	0.000	0.0
0.167	0.005	0.083	1.0
0.333	0.037	0.333	2.0
0.500	0.125	0.750	3.0
0.667	0.296	1.333	4.0
0.833	0.579	2.083	5.0
1.000	1.000	3.000	6.0
0.000	0.000	0.000	0.0
0.167	0.001	0.019	0.333
0.333	0.012	0.148	1.333
0.500	0.063	0.500	3.000
0.667	0.197	1.185	5.330
0.833	0.482	2.315	8.330
1.000	1.000	4.000	12.00

/

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```
ROHN SSV TOWER
0000.      2.27      1.000E11
0120.      2.27      7.380E10
0240.      2.00      4.750E10
0360.      2.00      3.100E10
0480.      2.00      1.450E10
0600.      1.97      5.870E+9
0720.      1.97      2.320E+9
  50.00       0
/
```

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Appendix B: The Rayleigh-Ritz Computer Code

The Rayleigh-Ritz computer code, which uses four assumed shape functions (four degrees of freedom) is listed below. This program has been written to accept input data directly from the terminal. The program could be modified to read data from an input file, however.

The tower must be divided into N segments, with constant values of mass and stiffness input along each segment. Emphasis must be placed on the fact that this program accepts only dimensionless values of stiffness and mass distribution $f(\xi)$ and $h(\xi)$, respectively. Also dimensionless values of top mass, guy wire attachment levels, and guy wire stiffness coefficients are input. These dimensionless properties were previously explained in Section 2.3.

The first quantity that must be input is N, the number of tower segments. For towers with highly nonlinear or tapered mass and stiffness distributions about twenty or thirty segments are suggested. Next, the top mass ratio μ must be input. After this, the guy wire properties are input. Provision has been made in the program to accept two sets of guy wires. The dimensionless guy wire attachment levels and stiffness coefficients are read in. These constants were defined in the nomenclature and Section 2.3.

In this program, numerical integration techniques are used to obtain the mass and stiffness coefficients for use in the Rayleigh-Ritz method. Numerical integration is performed over each tower segment. Each segment must be subdivided into a number of subsegments for numerical integration. The number of subsegments is denoted by DIV(I) for I = 1,2,..., N segments. Emphasis should be placed on the fact that DIV(I) should be a number divisable by four. The use of DIV(I)=12 is suggested.

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H(I) represents the dimensionless mass distribution value for a particular segment I. Recall from Section 2.3 that the dimensionless mass distribution $h(\xi)$ was obtained by:

$$h(\xi) = \frac{\rho A(\xi)}{\rho A_0}$$

i.e., by dividing by the base value of mass distribution. For each tower segment, a constant value of H(I) is input for the segment. It is suggested that for a particular segment the average value of mass distribution be input (the average value of dimensionless mass distribution of the values at the two endpoints).

F(I) represents the dimensionless stiffness distribution value for a particular segment I. The dimensionless stiffness distribution $f(\xi)$ is obtained by:

$$f(\xi) = \frac{EI(\xi)}{EI_0}.$$

Again it is suggested that for a particular tower segment, F(I) should represent the average of the values at the endpoints of that segment.

The quantity X(I + 1) represents the dimensionless position of the top endpoint of each segment, for I = 1,2,..., N. For example, if a twenty foot tower is being analyzed and is divided into four segments: X(1) = 0.00, X(2) = 0.25, X(3) = 0.50, X(4) = 0.75, and X(5) = 1.00.

Next, the quantities NMODE and EPS must be input. NMODE is the number of bending frequency ratios to be determined; while EPS is the error tolerance which determines when the convergence to the desired result has occurred. A suggested value for EPS is 0.01.

Also, the values WR and ADD must be input. WR is an estimate of the desired bending frequency ratio from below.

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If the first bending frequency ratio is being sought, WR must be a number less than this value. If the second bending frequency is being sought, WR must be a value less than this value, but greater than the first frequency ratio. The method used in this program will cause convergence to the first frequency ratio which is greater than WR.

The quantity ADD is a small incremental value which is used to increase WR and to test to see if this new value is closer to the desired frequency ratio. A suggested value for ADD is 0.5. It is suggested that in order to find the first bending frequency ratio, WR be set to 0.1. Once this frequency ratio has been determined and printed out, the user should set WR to some value somewhat greater than this first frequency ratio. The program should then proceed to converge to the second frequency ratio.

Two other values must be read into the program: IMOD and JMOD. These are values which determine whether or not a modeshape is to be calculated for a particular frequency. If IMOD and JMOD are both set equal to one, the program will calculate a modeshape corresponding to the frequency ratio just calculated and print the results on a local file named "SHAPE." If IMOD and JMOD are both set to values other than one, a modeshape will not be calculated.

Emphasis must be placed on the fact that the frequency ratios W, calculated by this program, are dimensionless quantities. The frequencies p are obtained from the frequency ratios W by:

$$p = \sqrt{\frac{EI_0}{\rho A_0 L^4}}\ W,$$

where EI_0, ρA_0, and L have been defined in the nomenclature and Section 2.1.

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```
      PROGRAM RITZ(INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT
     1,SHAPE,TAPE1=SHAPE,DEBUG=OUTPUT)
      DIMENSION DIV(301),H(300),F(300),X(400),Y2(4,301),Y0(4,301)
      DIMENSION Y1(4,301),HDIV(301),G(301),XNR(20),W(20,20)
      DIMENSION AIJ(4,4),BIJ(4,4),CIJ(4,4),ASUM(4,4),BSUM(4,4),
     1CSUM(4,4),A(301),B(301),C(301),FHI(4),GYSUM1(4,4),GYSUM2(4,4)
      DIMENSION SHAPE(50)
      INTEGER DIV
C MODIFIED FOR TOWER ANALYSIS INCLUDING GUY EFFECTS
      DO 10 II=1,4
      DO 10 J=1,4
      ASUM(II,J)=0.
      BSUM(II,J)=0.
      CSUM(II,J)=0.
   10 CONTINUE
C
      READ*,N,ALPHA,U
      READ*,RKC1,RKC2,XGUY1,XGUY2
      X(1)=0.
      AL=1./FLOAT(N)
      DO 20 I=1,N
      READ*,DIV(I),H(I),F(I),X(I+1)
      GO TO 18
      X(I+1)=X(I)+AL
      H(I)=1.-0.*X(I)
      F(I)=1.-.90*X(I)
      DIV(I)=4
   18 FDIV=FLOAT(DIV(I))
   20 HDIV(I)=(X(I+1)-X(I))/FDIV
C
      DO 170 I=1,N
      MI=DIV(I)
      MI1=MI+1
      G(1)=0.
      II=I
      DO 30 K=II,N
      XK1=X(K+1)
      XK=X(K)
      G(1)=G(1)+DIFINT(XK1,XK,ALPHA,H,K)
   30 CONTINUE
      G(1)=G(1)+U*(1.+ALPHA)
      DO 40 J=1,MI1
      HJ=FLOAT(J-1)
      XIN=X(I)+HJ*HDIV(I)
      IF(J.EQ.1)GO TO 35
      G(J)=G(J-1)-DIFINT(XIN,YIN1,ALPHA,H,I)
   35 Y2(1,J)=Y12(XIN)
      Y2(2,J)=Y22(XIN)
      Y2(3,J)=Y32(XIN)
      Y2(4,J)=Y42(XIN)
```


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```
      Y1(1,J)=Y11(XIN)
      Y1(2,J)=Y21(XIN)
      Y1(3,J)=Y31(XIN)
      Y1(4,J)=Y41(XIN)
      Y0(1,J)=Y10(XIN)
      Y0(2,J)=Y20(XIN)
      Y0(3,J)=Y30(XIN)
      Y0(4,J)=Y40(XIN)
      IF(XIN.NE.1.)GO TO 38
      PHI(1)=Y10(XIN)
      PHI(2)=Y20(XIN)
      PHI(3)=Y30(XIN)
      PHI(4)=Y40(XIN)
   38 XIN1=XIN
C
   40 CONTINUE
C
      HDIVI=HDIV(I)
      DO 150 II=1,4
      DO 150 J=1,4
      IF(J.LT.II)GO TO 150
      DO 120 K=1,MI1
      A(K)=YI2YJ2(I,II,J,K,F,Y2)
      C(K)=YI1YJ1(I,II,J,K,G,Y1)
  120 B(K)=YIYJ(I,II,J,K,H,Y0)
C
      SUMA1=SUMA2=SUMB1=SUMB2=SUMC1=SUMC2=SUMA3=SUMB3=SUMC3=0.
      DO 130 L1=2,MI,2
      SUMA1=SUMA1+A(L1)
      SUMB1=SUMB1+B(L1)
  130 SUMC1=SUMC1+C(L1)
      M1=MI-1
      DO 140 L2=3,M1,4
      SUMA2=SUMA2+A(L2)
      SUMB2=SUMB2+B(L2)
  140 SUMC2=SUMC2+C(L2)
      M3=MI-3
      IF(MI.EQ.4)GO TO 147
      DO 145 L3=5,M3,4
      SUMA3=SUMA3+A(L3)
      SUMB3=SUMB3+B(L3)
  145 SUMC3=SUMC3+C(L3)
C
  147 AIJ(II,J)=2.*HDIVI/45.*(7.*A(1)+32.*SUMA1+12.*SUMA2+
     114.*SUMA3+7.*A(MI+1))
      BIJ(II,J)=2.*HDIVI/45.*(7.*B(1)+32.*SUMB1+12.*SUMB2
     1+14.*SUMB3+7.*B(MI+1))
      CIJ(II,J)=2.*HDIVI/45.*(7.*C(1)+32.*SUMC1+12.*SUMC2
     1+14.*SUMC3+7.*C(MI+1))
  150 CONTINUE
      DO 160 II=1,4
```


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```
      DO 160 J=1,4
      IF(J.LT.II)GO TO 160
      ASUM(II,J)=ASUM(II,J)+AIJ(II,J)
      BSUM(II,J)=BSUM(II,J)+BIJ(II,J)
      CSUM(II,J)=CSUM(II,J)+CIJ(II,J)
      CSUM(J,II)=CSUM(II,J)
      ASUM(J,II)=ASUM(II,J)
      BSUM(J,II)=BSUM(II,J)
  160 CONTINUE
  170 CONTINUE
      IF(U.EQ.0.)GO TO 175
      DO 165 II=1,4
      DO 165 J=1,4
      PHI2=PHI(II)*PHI(J)
      BSUM(II,J)=BSUM(II,J)+U*PHI2
  165 CONTINUE
  175 IF(RKC1.EQ.0.)GO TO 178
      Y1GUY1=Y10(XGUY1)
      Y2GUY1=Y20(XGUY1)
      Y3GUY1=Y30(XGUY1)
      Y4GUY1=Y40(XGUY1)
      GYSUM1(1,1)=Y1GUY1**2
      GYSUM1(1,2)=Y1GUY1*Y2GUY1
      GYSUM1(1,3)=Y1GUY1*Y3GUY1
      GYSUM1(1,4)=Y1GUY1*Y4GUY1
      GYSUM1(2,2)=Y2GUY1**2
      GYSUM1(2,3)=Y2GUY1*Y3GUY1
      GYSUM1(2,4)=Y2GUY1*Y4GUY1
      GYSUM1(3,3)=Y3GUY1**2
      GYSUM1(3,4)=Y3GUY1*Y4GUY1
      GYSUM1(4,4)=Y4GUY1**2
      GYSUM1(2,1)=GYSUM1(1,2)
      GYSUM1(3,1)=GYSUM1(1,3)
      GYSUM1(3,2)=GYSUM1(2,3)
      GYSUM1(4,1)=GYSUM1(1,4)
      GYSUM1(4,2)=GYSUM1(2,4)
      GYSUM1(4,3)=GYSUM1(3,4)
C
      Y1GUY2=Y10(XGUY2)
      Y2GUY2=Y20(XGUY2)
      Y3GUY2=Y30(XGUY2)
      Y4GUY2=Y40(XGUY2)
      GYSUM2(1,1)=Y1GUY2**2
      GYSUM2(1,2)=Y1GUY2*Y2GUY2
      GYSUM2(1,3)=Y1GUY2*Y3GUY2
      GYSUM2(1,4)=Y1GUY2*Y4GUY2
      GYSUM2(2,2)=Y2GUY2**2
      GYSUM2(2,3)=Y2GUY2*Y3GUY2
      GYSUM2(2,4)=Y2GUY2*Y4GUY2
      GYSUM2(3,3)=Y3GUY2**2
      GYSUM2(3,4)=Y3GUY2*Y4GUY2
```

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```
      GYSUM2(4,4)=Y4GUY2**2
      GYSUM2(2,1)=GYSUM2(1,2)
      GYSUM2(3,1)=GYSUM2(1,3)
      GYSUM2(4,1)=GYSUM2(1,4)
      GYSUM2(3,2)=GYSUM2(2,3)
      GYSUM2(4,2)=GYSUM2(2,4)
      GYSUM2(4,3)=GYSUM2(3,4)
C
C
C FIND THE ROOTS OF THE DETERMINANT
C USE TRIAL VALUES OF OMEGA
C
C$    ARRAYS
C$    FUNCS
C$    TRACE
C$    CALLS
  178 READ*,NMODE,NROTS,EPS
      DO 600 I=1,NMODE
      READ*,OMEGA,WR,ADD
      READ*,IMOD,JMOD
      WL=WR
      POLY1=POLYN(WL,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1,GYSUM2)
  200 POLY2=POLYN(WL+ADD,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1,GYSUM2)
      IF(POLY1*POLY2.LT.0.)GO TO 300
      WL=WL+ADD
      POLY1=POLY2
      GO TO 200
  300 WR=WL+ADD
C NOW USE THE SECANT METHOD
      WR1=WL
      WR2=WR
      POLY3=POLY2
      DO 600 J=1,NROTS
      ITS=0
  400 POLYD=POLY1-POLY2
      DELTA=WR1-WR2
      WR1=WR2
      WR2=WR2-DELTA*POLY2/POLYD
      POLY3=POLY2
      POLY1=POLY2
      POLY2=POLYN(WR2,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1,GYSUM2)
      ERR=WR1-WR2
      ERROR=ABS(ERR)
      ITS=ITS+1
      IF(ITS.GT.50)STOP
      IF(ERROR.GT.EPS)GO TO 400
      PRINT*,WR2
      W(I,J)=WR2
      XNR(J)=OMEGA
      IF(J.NE.JMOD)GO TO 450
      IF(I.NE.IMOD)GO TO 450
```


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```
      CALL EIGEN(SHAPE,X,WR2,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1
     1,GYSUM2)
      DO 48 K=1,51
      WRITE(1,45)X(K),SHAPE(K)
   45 FORMAT(2F10.4)
   48 CONTINUE
  450 OMEGA=OMEGA+1.
      IF(J.EQ.NROTS)GO TO 600
      POLY1=POLYN(WR1,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1,GYSUM2)
      POLY2=POLYN(WR2,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1,GYSUM2)
  600 CONTINUE
  650 STOP
      END
C
C
      FUNCTION DIFINT(X1,X2,A,H,I)
      DIMENSION H(300)
      DIFINT=H(I)*(A*(X1-X2)+.5*(X1**2-X2**2))
      RETURN
      END
C
C
      FUNCTION Y11(E)
      Y11=4.*E**3-12.*E**2+12.*E
      RETURN
      END
C
C
      FUNCTION Y21(E)
      Y21=15.*E**4-40.*E**3+30.*E**2
      RETURN
      END
C
C
      FUNCTION Y31(E)
      Y31=12.*E**5-30.*E**4+20.*E**3
      RETURN
      END
C
C
      FUNCTION Y41(E)
      Y41=10.*E**9-22.5*E**8+12.856*E**7
      RETURN
      END
C
      FUNCTION Y10(E)
      Y10=E**4-4.*E**3+6.*E**2
      RETURN
      END
C
C
```

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```
C
C
      FUNCTION Y20(E)
      Y20=3.*E**5-10.*E**4+10.*E**3
      RETURN
      END
C
C
      FUNCTION Y30(E)
      Y30=2.*E**6-6.*E**5+5.*E**4
      RETURN
      END
C
C
      FUNCTION Y40(E)
      Y40=E**10-2.5*E**9+1.607*E**8
      RETURN
      END
C
C
      FUNCTION Y12(E)
      Y12=12.*E**2-24.*E+12.
      RETURN
      END
C
C
      FUNCTION Y22(E)
      Y22=60.*E**3-120.*E**2+60.*E
      RETURN
      END
C
C
      FUNCTION Y32(E)
      Y32=60.*E**4-120.*E**3+60.*E**2
      RETURN
      END
C
C
      FUNCTION Y42(E)
      Y42=90.*E**8-180.*E**7+89.992*E**6
      RETURN
      END
C
C
      FUNCTION YI2YJ2(I,II,J,K,F,Y2)
      DIMENSION F(300),Y2(4,301)
      YI2YJ2=Y2(II,K)*Y2(J,K)*F(I)
      RETURN
      END
C
C
```


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```
      FUNCTION YI1YJ1(I,II,J,K,G,Y1)
      DIMENSION G(301),Y1(4,301)
      YI1YJ1=Y1(II,K)*Y1(J,K)*G(K)
      RETURN
      END
C
C
      FUNCTION YIYJ(I,II,J,K,H,Y0)
      DIMENSION H(300),Y0(4,301)
      YIYJ=Y0(II,K)*Y0(J,K)*H(I)
      RETURN
      END
      FUNCTION POLYN(WI,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1,GYSUM2)
      DIMENSION B(5,5),ASUM(4,4),BSUM(4,4),CSUM(4,4),D(5)
      DIMENSION GYSUM1(4,4),GYSUM2(4,4)
      OMEGA2=OMEGA*OMEGA
      W2=WI*WI
      DO 200 I=1,4
      DO 200 J=1,4
      B(I,J)=ASUM(I,J)+OMEGA2*(CSUM(I,J)-W2*BSUM(I,J)+RKC1*GYSUM1(I,J)
     1+RKC2*GYSUM2(I,J)
  200 CONTINUE
      POLYN=1.
      DO 500 I=1,4
      L=1
      DO 450 J=1,4
      AJ=ABS(B(I,J))
      AL=ABS(B(I,L))
      IF(AJ.LE.AL)GO TO 450
      L=J
  450 CONTINUE
      AMAX=B(I,L)
      DO 460 J=1,4
  460 B(I,J)=B(I,J)/AMAX
      POLYN=AMAX*POLYN
  500 CONTINUE
   70 DO 300 K=1,4
      KP=K+1
      IF(K.EQ.4)GO TO 80
      M=K
      DO 55 I=KP,4
      AI=ABS(B(I,K))
      AM=ABS(B(M,K))
      IF(AM.GE.AI)GO TO 55
      M=I
   55 CONTINUE
      IF(M.EQ.K)GO TO 80
   60 DO 65 J=K,4
      D(J)=B(K,J)
      B(K,J)=B(M,J)
      B(M,J)=D(J)
```


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```
   65 CONTINUE
      POLYN=-1.*POLYN
   80 POLYN=B(K,K)*POLYN
      IF(POLYN.NE.0.)GO TO 90
      GO TO 380
   90 DO 300 J=KP,4
      B(K,J)=B(K,J)/B(K,K)
      DO 300 I=KP,4
  300 B(I,J)=B(I,J)-B(K,J)*B(I,K)
  380 RETURN
      END
      SUBROUTINE EIGEN(SHAPE,X,WR2,OMEGA,ASUM,BSUM,CSUM,RKC1,RKC2,GYSUM1
     1,GYSUM2)
      DIMENSION SHAPE(50),X(400),ASUM(4,4),BSUM(4,4),CSUM(4,4)
     1,B(3,4),D(4),DYN(4,4),C(3),A(4)
      DIMENSION GYSUM1(4,4),GYSUM2(4,4)
      OMEGA2=OMEGA*OMEGA
      W2=WR2*WR2
      DO 20 I=2,4
      DO 20 J=1,4
      DYN(I,J)=ASUM(I,J)+OMEGA2*CSUM(I,J)-W2*BSUM(I,J)+RKC1*GYSUM1(I,J)
     1+RKC2*GYSUM2(I,J)
   20 CONTINUE
C
      DO 30 I=1,3
      DO 30 J=1,3
   30 B(I,J)=DYN(I+1,J+1)
      B(1,4)=-DYN(2,1)
      B(2,4)=-DYN(3,1)
      B(3,4)=-DYN(4,1)
C
      DO 500 I=1,3
      L=1
      DO 450 J=1,3
      AJ=ABS(B(I,J))
      AL=ABS(B(I,L))
      IF(AJ.LE.AL)GO TO 450
      L=J
  450 CONTINUE
      AMAX=B(I,L)
      DO 460 J=1,4
  460 B(I,J)=B(I,J)/AMAX
  500 CONTINUE
C PARTIAL PIVOTING
   70 DO 300 K=1,3
      KP=K+1
      IF(K.EQ.3)GO TO 90
      M=K
      DO 45 I=KP,3
      AI=ABS(B(I,K))
      AM=ABS(B(M,K))
```


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```
      IF(AM.GE.AI)GO TO 45
      M=I
   45 CONTINUE
      IF(M.EQ.K)GO TO 90
   60 DO 65 J=K,4
      D(J)=B(K,J)
      B(K,J)=B(M,J)
      B(M,J)=D(J)
   65 CONTINUE
   90 DO 300 J=KP,4
      B(K,J)=B(K,J)/B(K,K)
      DO 300 I=KP,3
      B(I,J)=B(I,J)-B(K,J)*B(I,K)
  300 CONTINUE
C BACK SOLUTION
      C(3)=B(3,4)
  310 DO 350 I=2,3
      k=3-(I-1)
      K1=K+1
      SUM=0.
      DO 320 J=K1,3
  320 SUM=SUM+B(K,J)*C(J)
      C(K)=B(K,4)-SUM
  350 CONTINUE
C
C FIND THE SHAPE
      ADD=1./50.
      X(1)=0.
      DO 600 I=2,51
      X(I)=X(I-1)+ADD
      DO 50 J=2,51
      XJ=X(J)
   50 SHAPE(J)=Y10(XJ)+C(1)*Y20(XJ)+C(2)*Y30(XJ)
     1+C(3)*Y40(XJ)
  600 CONTINUE
      A(1)=1.
      A(2)=C(1)
      A(3)=C(2)
      A(4)=C(3)
      PRINT*,A(1),A(2),A(3),A(4)
      SUMA=SUMB=SUMGY1=SUMGY2=0.
      DO 110 I=1,4
      DO 110 J=1,4
      SUMA=SUMA+A(I)*A(J)*ASUM(I,J)
      SUMB=SUMB+A(I)*A(J)*BSUM(I,J)
      SUMGY1=SUMGY1+A(I)*A(J)*GYSUM1(I,J)
  110 SUMGY2=SUMGY2+A(I)*A(J)*GYSUM2(I,J)
      AN=SUMA/SUMB
      BN1=SUMGY1/SUMB
      BN2=SUMGY2/SUMB
      PRINT*,AN,BN1,BN2
```

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```
      SHAPE(1)=0.
      DO 55 I=1,51
   55 SHAPE(I)=SHAPE(I)/SHAPE(51)
  700 RETURN
      END
/
```


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Program Record 101

TI Programmable 59

"ISCOE"

INERTIA AND STIFFNESS COEFFICIENTS
OF WIND TURBINE SYSTEMS

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and
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Program Abstract

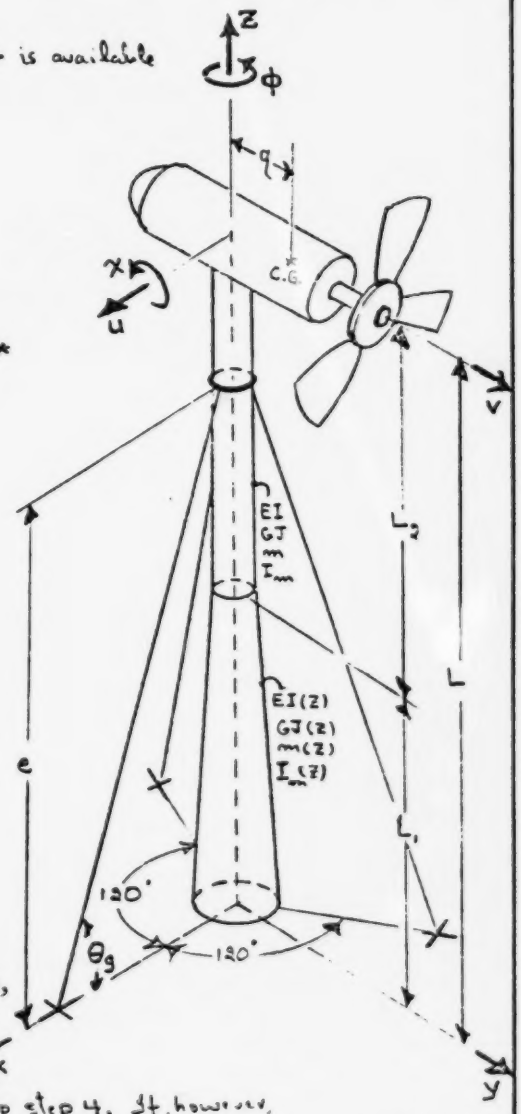
Program Title: INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES | P. 1

This program evaluates the inertia and the stiffness coefficients of wind turbine systems.

The figure below depicts a wind turbine system with a symmetrical rotor and a guy stiffened tower. The tower has a tapered section of length L_1, and a uniform section of length L_2. The base of the tower may be fixed (cant. lever beam tower) or it may be simply supported (balljoint base constrained against rotation). The linear equations of unforced motion for this system are

$$\begin{bmatrix} M_{uu} & 0 & M_{u\phi} & 0 \\ 0 & M_{vv} & 0 & M_{vx} \\ M_{u\phi} & 0 & M_{\phi\phi} & 0 \\ 0 & M_{vx} & 0 & M_{xx} \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{\phi} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega I_r \\ 0 & 0 & -\Omega I_r & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{\phi} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} k_{uu} & 0 & 0 & 0 \\ 0 & k_{vv} & 0 & k_{vx} \\ 0 & 0 & k_{\phi\phi} & 0 \\ 0 & k_{vx} & 0 & k_{xx} \end{bmatrix} \begin{Bmatrix} u \\ v \\ \phi \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

To evaluate the inertia coefficients, M_{ij} and the stiffness coefficients, k_{ij}, the following procedure is available



1. SELECT TYPE OF TOWER BASE.

   For cantilever beam tower, press A.
   For ball joint base, press A'.

2. ENTER DATA FOR GUY CABLES.*

   Press B; Input: L, k_c, e.

3. ENTER DATA FOR NON-UNIFORM SECTION L_1.**

   Press C'; Input: L, $L'=L_1$, N.
   Input: $EI(z)$, $GJ(z)$, $m(z)$, $I_m(z)$
   for $z = 0, \frac{L'}{N}, \frac{2L'}{N}, \frac{3L'}{N}, \ldots, L'$.

4. ENTER DATA FOR UNIFORM SECTION L_2.

   Press C; Input: L, $L'=L_2$, N.
   Input: EI, GJ, m, I_m only once for $z=L_1$.

5. ENTER DATA FOR ROTOR & NACELLE.

   Press D; Input: I_{xx}, I_{zz}, m_r+m_n, q.

6. PRINT RESULTS.

   Press E; Output: M_{uu}, $M_{u\phi}$, M_{vv}, M_{vx}, $M_{\phi\phi}$, M_{xx}, k_{uu}, k_{vv}, k_{vx}, $k_{\phi\phi}$, k_{xx}.

* Repeat step 2 for each additional set of 3 cable guys.

** If the entire tower is non-uniform, enter $L'=L$ and skip step 4. If, however, the entire tower has constant properties, then skip step 3 and perform step 4 with $L'=L$.

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User Instructions

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINE	p. 2

STEP	PROCEDURE	PRINTOUT PC-100A	DISPLAY
	LOAD CARD SIDES 1, 2, AND 3.		
1	SELECT TYPE OF TOWER BASE & INITIALIZE.		
	Perform step 1a or 1b.		
1a	Press A for cantilever beam tower.	00	
1b	Press A' for ball joint base.	0*	0
2	ENTER DATA FOR GUY CABLES. (L, k_c, a) Skip to step 3 if tower is not guyed.		
2a	Press B.	L	
2b	Enter L, tower length.	ll.ll	ll.ll
2c	Press R/S.	Kc	
2d	Enter k_c, effective guy stiffness.	kk.kk	kk.kk
2e	Press R/S.	E	
2f	Enter e, height of cable attachment point.	ee.ee	ee.ee
2g	Press R/S		
2h	Wait for response characters 00 or 0* which indicate tower base type selected in step 1.	00 or 0*	29
2i	Repeat step 2 for each additional set of 3 cables.		
3	ENTER DATA FOR TOWER (L, L', N, EI, GJ, m, I_m)		
	Perform step 3a or 3b.		
3a	Press C' for section with variable properties		
3b	Press C for uniform tower sections.	L	
3c	Enter L, tower length.	ll.ll	ll.ll
3d	Press R/S.	L'	
3e	Enter L', section length.	ll.ll'	ll.ll'
3f	Press R/S.	N	
3g	Enter N, even number of intervals for integration.	nn	nn
3h	Press R/S		
3i	Wait for coordinate Z to be printed.	Z = zz.zz	
3j	Enter EI(Z), bending stiffness of tower at Z.	EI ei.ei	ei.ei
3k	Press R/S.	GJ	
3l	Enter GJ(Z), torsional stiffness of tower at Z.	gj.gj	gj.gj
3m	Press R/S.	M	
3n	Enter m(Z), mass per unit length at Z.	mm.mm	mm.mm
3o	Press R/S	I M	
3p	Enter I_m(Z), mass moment of inertia per unit length about the center line of tower.	im.im	im.im

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Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES | P 3

STEP	PROCEDURE	PRINTOUT PC-100A	DISPLAY
3q	Press R/S.		
3r	If C' was pressed (step 3a) for variable section properties, then steps 3i through 3q have to be repeated until the data for the entire section L' has been entered. If C was pressed (step 3b), wait for program to complete computations. (approximately 110 seconds for N=10). Response upon completion is again 00 or 0*.	00 or 0*	
4	REPEAT STEP 3 FOR NEXT SECTION.		
5	ENTER ROTOR & NACELLE DATA (I_{xx}, I_{zz}, m_r+m_n, q)		
5a	Press D.	I XX	
5b	Enter I_{xx}, rotor & nacelle mass moment of inertia about x.	xx.xx	xx.xx
5c	Press R/S.	I ZZ	
5d	Enter I_{zz}, rotor & nacelle mass moment of inertia about z.	zz.zz	zz.zz
5e	Press R/S.	MR+MN	
5f	Enter m_r+m_n, total mass of rotor & nacelle.	rr.rr	rr.rr
5g	Press R/S.	Q	
5h	Enter q, the distance between the center of gravity of the rotor & nacelle and the tower center line	qq.qq	qq.qq
5i	Press R/S.	00	
5j	Wait for response characters 00 or 0*.	or 0*	
6	PRINT INERTIA AND MASS COEFFICIENTS.		
6a	Press E.	M UU	
6b	Wait for a complete printout of results followed by response characters 00 or 0*.	uu.uu M UQ φφ.φφ • • • K XX xx.xx	
	Note: Step 6 may be performed at any stage of the run following completion of steps 1, 2, 3, 4, or 5, if intermediate results are desired.		
	Note: For a guy stiffened tower with a rotor and nacelle, steps 1, 2, 3, and 4 must be performed in sequence.	00 or 0*	

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User Information

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES	P. 4

USER DEFINED KEYS

A - Cantilever beam tower.
A' - Ball joint base tower.
B - Enter L, k_c, e.
B' - ~
C - Enter L, L', N, EI, GJ, m, I_m
C' - Enter $L, L', N, EI(z), GJ(z), m(z), I_m(z)$
D - Enter I_{xx}, I_{zz}, m_r+m_n, q
D' - ~
E - Print results → $M_{uu}, M_{u\phi}, \ldots, K_{xx}$
E' - Used.

☑ t Register — Partition 479,59 **
☑ Absolute Addresses — Program Steps 480
☑ PC - 100A* — Library Module —
☑ Disturbs Pending Operations — Card Sides 3
Angular Mode —
Parentheses Level — — SBR Levels 2

* A printer will be very useful but is not essential (see p.10)
** Initial partition for TI 59

Flags	0 Output?	1 Initialize?	2	3	4
	5	6	7	8	9

* DATA REGISTERS *

Contents	Reg.	Comments	Contents	Reg.	Comments
Ψ_ϕ; Counter	00	Computed	101 ; 159	30	SBR address
Ψ_v; pointer	01	"	Δz	31	Computed
Ψ_z; pointer	02	"	$\Delta z/3$; $2\Delta z/3$	32	"
Ψ_v"	03	"	$4\Delta z/3$	33	"
Ψ_x"	04	"	Integrands	34	"
L	05	Input	2700. " L "	35	Label
L'	06	"	2765. " L' "	36	"
N	07	"	3100. " N "	37	"
z	08	Computed	4664. " Z= "	38	"
EI	09	Input	1724. " EI "	39	"
GJ	10	"	2225. " GJ "	40	"
m	11	"	3000. " M "	41	"
I_m	12	"	2430. " IM "	42	"
k_c	13	"	260015. " K C "	43	"
e	14	"	.1700. " E "	44	"
I_{xx}	15	"	24004444. " I XX "	45	"
I_{zz}	16	"	24004646. " I ZZ "	46	"
m_r+m_n	17	"	3035473031. "MR+MN"	47	"
q; SBR address	18	"	3400. " Q "	48	"
M_{uu}	19	Computed	30004141. " M uu "	49	"
$M_{u\phi}$	20	"	30004134. " M uQ "	50	"
M_{vv}	21	"	30004242. " M vv "	51	"
M_{vx}	22	"	30004244. " M vx "	52	"
$M_{\phi\phi}$	23	"	30003434. " M QQ "	53	"
M_{xx}	24	"	30004444. " M xx "	54	"
k_{uu}	25	"	26004141. " K uu "	55	"
k_{vv}	26	"	26004242. " K vv "	56	"
k_{vx}	27	"	26004244. " K vx "	57	"
$k_{\phi\phi}$	28	"	26003434. " K QQ "	58	"
k_{xx}	29	"	26004444. " K xx "	59	"

Notes:

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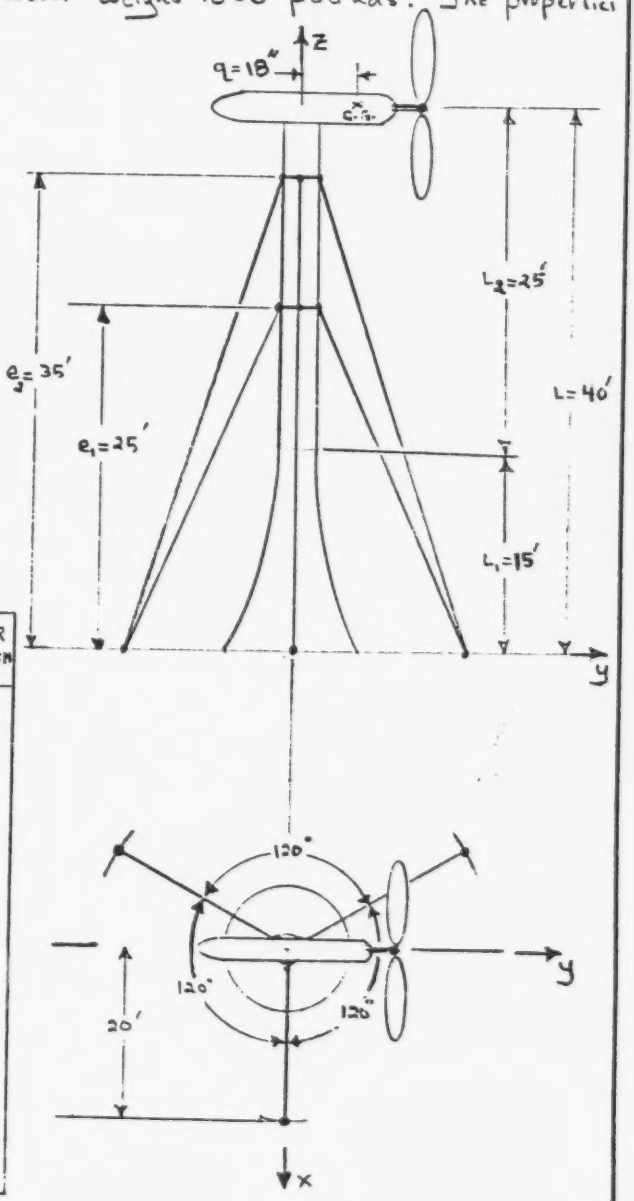
Sample Problem

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES | P 5

Find the inertia coefficients, M_{ij}, and the stiffness coefficients, K_{ij}, for the wind turbine system shown below. The tower base is a ball joint design which is constrained to prevent rotation. The tower is 40 ft in length and is stiffened by two sets of three 1/2 in. diameter cables installed 120° apart on a 20 ft radius circle. The first set is attached to the tower at a height of 25 ft and the second set at 35 ft. The rotor and nacelle system atop the tower weighs 1800 pounds. The properties and specifications are as follows:



GUYS:

1/2" dia cables with

$A = .124\ in^2$ $\quad E = 20 \times 10^6\ psi$

$e_1 = 25\ ft$ $\quad e_2 = 35\ ft$

$l_1 = 32.01\ ft$ $\quad l_2 = 40.31\ ft$

$\cos\theta_{g_1} = .625$ $\quad \cos\theta_{g_2} = .496$

$K_{c_1} = 45368\ lb/ft$ $\quad K_{c_2} = 22718\ lb/ft$

TOWER:

Ball joint base (no rotation) L = 40 ft

with properties tabulated below

z ft	EI lb ft²	GJ lb ft²	m lb s²/ft/ft	I m lb s² ft/ft	TOWER SECTION
0.0	10.5×10^6	7.1×10^6	.3765	90.562	
1.5	9.7×10^6	6.5×10^6	.3632	83.321	
3.0	8.3×10^6	5.5×10^6	.3523	78.697	
4.5	7.1×10^6	4.7×10^6	.3315	67.524	
6.0	6.2×10^6	4.1×10^6	.2935	53.356	
7.5	5.6×10^6	3.7×10^6	.2613	50.872	L_1
9.0	4.5×10^6	3.1×10^6	.2201	45.911	
10.5	3.2×10^6	2.1×10^6	.1982	40.654	
12.0	2.1×10^6	1.4×10^6	.1534	28.852	
13.5	1.9×10^6	1.3×10^6	.1356	20.951	
15.0	1.5×10^6	1.1×10^6	.1245	15.296	
40.0	"	"	"	"	L_2

ROTOR & NACELLE:

$I_{xx} = I_{zz} = 550\ lb\ s^2\ ft$

$m_r + m_n = 1800\ lb = 55.9\ lb\ s^2/ft$.

$q = 18\ in = 1.5\ ft$.

Sample Run

Program Title: INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES					P.6
ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
	Load card sides 1, 2, and 3. (See note 1 below)				
	A'	0-	INITIALIZE AND SELECT BALL - JOINT BASE	0.	response time is approximately 8 seconds
	B	L	GUY 1 DATA	0.	
40	R/S	40.			
		K C		0.	
45368	R/S	45368.			
		E.		0.	
25	R/S	25.			
		0*	response characters for ball joint base	29.	response takes approx. 6 secs.
	B	L	GUY 2 DATA	40.	see note 2
	R/S	40.			
		K C		45368.	"
22718	R/S	22718.			
		E		25.	"
35	R/S	35.			
		0*	If intermediate results are desired, press E		
	C'		TOWER DATA NON-UNIFORM SECTION	40.	see note 2
		L			
	R/S	40.			
		L'		0.	
15	R/S	15.			
		N		0.	
10	R/S	10.			
		Z=	coordinate Z		
		0.			
		EI		0.	
10500000	R/S	10500000.	see note 3		
		GJ		0.	
7100000	R/S	7100000.			
		M		0.	
.3765	R/S	0.3765			
		IM		0.	
90.562	R/S	90.562			
	Wait for completion of computations (approximately 10 seconds). Response is a printout of Z				

Note 1: All the quantities in bank 4 are computed or input values. There is therefore no need to load card side 4, unless some frequently used input values have been saved on bank 4 (see note 2).

Note 2: Before each data entry, the program displays the register content. If no change is desired the user simply presses R/S.

Note 3: If scientific notation is used, it must be removed before pressing R/S eg. 10.5 EE 6 INV EE R/S

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Sample Run

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES					P.7
ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
		Z=	Coordinate Z		
		1.5			
		EI		10500000	
9700000	R/S	9700000.			
		GJ		7100000	
6500000	R/S	6500000.			
		M		.7365	
		0.3632			
		IM		79.562	
		83.321			
		Z=	coordinate Z		
		3.			
		EI		9700000	
8300000	R/S	8300000.			
		GJ		6500000	
5500000	R/S	5500000.			
		M		.3632	
.3523	R/S	0.3523			
		IM		83.321	
78.697	R/S	78.697			
		Z=	Coordinate Z		
		4.5			
		EI		8300000	
7100000	R/S	7100000.			
		GJ		5500000	
4700000	R/S	4700000.			
		M		.3632	
.3315	R/S	0.3315			
		IM		78.697	
67.524	R/S	67.524			
		Z=	Coordinate Z		
		6.			
		EI		7100000	
6200000		6200000.			
		GJ		4700000	
41	R/S(oops!)	~~41.~~ 4100000	see note 1		
4100000	STO 10	M		.3315	
.2935	R/S	0.2935			
		IM		67.524	
53.356	R/S	53.356			

Note 1: To recover from erroneous inputs after R/S, key in correct input and store directly.
EI is STO 09 : GJ is STO 10 : M is STO 11.
To correct IM requires program interrupt and is not recommended.

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Sample Run

Program Title: INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINE.					P.8
ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
		Z= 7.5	coordinate Z		
		EI		6200000	
5600000	R/S	5600000.			correction for GJ at z=6 was successful!
		GJ		4100000	
3700000	R/S	3700000.			
		M		.2935	
.2613	R/S	0.2613			
		IM		53.356	
50.872	R/S	50.872			
		Z= 9.	coordinate Z		
		EI		5600000	
4500000	R/S	4500000.			
		GJ		3700000	
3100000	R/S	3100000.			
		M		.2613	
.2201	R/S	0.2201			
		IM		50.872	
45.911	R/S	45.911			
		Z= 10.5	coordinate Z		
		EI		4500000	
3200000	R/S	3200000.			
		GJ		3100000	
2100000	R/S	2100000.			
		M		.2201	
.1982	R/S	0.1982			
		IM		45.911	
40.654	R/S	40.654			
		Z= 12.	coordinate Z		
		EI		3200000	
2100000	R/S	2100000.			
		GJ		2100000	
1400000	R/S	1400000.			
		M		.1982	
.1534	R/S	0.1534			
		IM		40.654	
28.852	R/S	28.852			

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Sample Run

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINE					P.9
ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
		Z=	coordinate Z		
		13.5			
				2100000	
		EI			
1900000	R/S	1900000.		1400000	
		GJ			
1300000	R/S	1300000.		.1534	
		M			
.1356	R/S	0.1356		28.852	
		IM			
20.951	R/S	20.951			
		Z=	coordinate Z		
		15.	end of section L_1		
				1900000	
		EI			
1500000	R/S	1500000.		1300000	
		GJ			
1100000	R/S	1100000.		.1356	
		M			
.1245	R/S	0.1245		20.951	
		IM			
15.896	R/S	15.896			
		0*		29	
	C		TOWER DATA UNIFORM SECT.		
		L		40.	
	R/S	40.			
		L'		15.	
25	R/S	25.			
		N		0.	
10	R/S	10.			
		Z=	coordinate Z begin at $Z=L_1$		
		15.			
				1500000.	
		EI			
	R/S	1500000.		1100000.	
		GJ			
	R/S	1100000.		.1245	
		M			
	R/S	0.1245		15.896	
		IM			
	R/S	15.896			
		0* Display 29	Computation time depends on N. The program takes approximately 110 seconds to complete the computations.		For N=10

Note : If desired, Press E before adding the nacelle and rotor system. The output will be the inertia coefficients, m_{ij} for the tower alone. The stiffness coefficients are unaffected by the addition of rotor and nacelle.

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Sample Run

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES					P.10
ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
	D		NACELLE & ROTOR DATA		
		I XX		0.	
550	R/S	550.			
		I ZZ		0.	
550	R/S	550.			
		MR+MN		0.	
55.9	R/S	55.9			
		0		393	this number is an SER address stored temporarily in register 18
1.5	R/S	1.5			
		0*		29	
	E		PRINT FINAL RESULTS		
		M UU			
		56.43326281			
		M UQ			
		-83.85			
		M VV			
		58.40760828			
		M VX			
		18.21937687			
		M QQ			
		774.3475923			
		M XX			
		716.5607592			
		K UU			
		6865.538931			
		K VV			
		51949.54887			
		K VX			
		375850.9686			
		K QQ			
		51437.5			
		K XX			
		3133349.291			
			END OF RUN		
		0*		0	

RUNNING THE PROGRAM WITHOUT A PC-100 PRINTER

To run this program without a printer, the user should proceed as follows:

1. Load card sides 1, 2, and 3
2. Edit the program by pressing the following key sequence.

GTO 5 7 LRN 2nd NOP 2nd NOP LRN .

With this modification, the program will halt and display output quantities. The user notes the display and presses R/S to resume program execution.

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Referring to the wind turbine system and the equations of motion given on page 1, the stiffness coefficients, k_{ij}, and the inertia coefficients, M_{ij}, are evaluated as follows

$$k_{vv} = \int_0^L EI(z)\,(\psi_v''(z))^2\,dz + k_c\,\psi_v^2(e)$$
$$k_{xv} = \int_0^L EI(z)\,\psi_v''(z)\,\psi_x''(z)\,dz + k_c\,\psi_v(e)\,\psi_x(e)$$
$$k_{xx} = \int_0^L EI(z)\,(\psi_x''(z))^2\,dz + k_c\,\psi_x^2(e)$$
$$k_{\phi\phi} = \int_0^L GJ(z)\,(\psi_\phi'(z))^2\,dz$$
$$k_{uu} = k_{vv} - k_{vx}^2/k_{xx}$$

$$M_{vv} = \int_0^L m(z)\,\psi_v^2(z)\,dz + (m_r + m_m)$$
$$M_{xv} = \int_0^L m(z)\,\psi_v(z)\,\psi_x(z)\,dz$$
$$M_{xx} = \int_0^L m(z)\,\psi_x^2(z)\,dz + I_{xx}$$
$$M_{\phi\phi} = \int_0^L I_m(z)\,\psi_\phi^2(z)\,dz + I_{zz}$$
$$M_{u\phi} = -(m_r + m_m)\,q$$
$$M_{uu} = M_{vv} - 2(k_{vx}/k_{xx})\,M_{vx} + (k_{vx}/k_{xx})^2\,(M_{xx} - I_{xx})$$

where,

L = Tower length.

EI = Tower bending stiffness.

GJ = Tower torsional stiffness.

k_c = Effective guy stiffness $= 3/2\,(AE/\ell)\cos^2\theta_g$

where

A = Cable cross sectional area.

E = Cable modulus of elasticity

ℓ = Cable length.

θ_g = Angle between guys and ground.

e = Height of cable attachment point.

m = Mass per unit length of tower.

I_m = Mass moment of inertia of tower per unit length.

$m_r + m_m$ = Total mass of rotor and nacelle.

I_{xx}, I_{zz} = Mass moments of inertia of rotor and nacelle about x and y.

and ψ_ϕ', ψ_v'', ψ_x'' are the derivatives with respect to z of the displacement functions ψ_i tabulated below for the cantilever beam tower and the ball joint base tower

CANTILEVER BEAM TOWER	BALL JOINT BASE (NO ROTATION)
$\psi_v(z) = (z/L)^2(3 - 2z/L)$	$\psi_v(z) = (1/2)(z/L)(3 - (z/L)^2)$
$\psi_x(z) = z(z/L)(1 - z/L)$	$\psi_x(z) = (z/2)(1 - (z/L)^2)$
$\psi_\phi(z) = (z/L)$	$\psi_\phi(z) = (z/L)$

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Program Listing

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES | P 1:

LOC.	CODE	KEY	COMMENTS
000	25	CLR	
001	42	STO	
002	01	01	
003	76	LBL	SBR INPUT,
004	10	E'	OUTPUT, OR
005	98	ADV	INITIALLIZE
006	25	CLR	
007	87	IFF	
008	01	01	Initialize?
009	00	00	
010	33	33	
011	43	RCL	Variable
012	01	01	pointer
013	85	+	
014	03	3	
015	00	0	
016	95	=	
017	42	STO	Label
018	02	02	pointer
019	73	RC*	Label
020	02	02	
021	69	DP	
022	02	02	
023	69	DP	Print
024	05	05	label
025	73	RC*	Display
026	01	01	variable
027	87	IFF	
028	00	00	R/S?
029	00	00	
030	32	32	
031	91	R/S	
032	99	PRT	Print
033	72	ST*	variable
034	01	01	Store
035	69	DP	entry
036	21	21	Move
037	97	DSZ	pointer
038	00	00	
039	00	00	
040	06	06	
041	92	RTN	
042	76	LBL	CANTILEVER
043	11	A	BEAM TOWER
044	01	1	
045	00	0	
046	01	1	
047	42	STO	SBR Ψ address
048	30	30	
049	25	CLR	
050	42	STO	Z
051	08	08	
052	86	STF	Initialize
053	01	01	(3ⁿᵈ M's & K's)

LOC.	CODE	KEY	COMMENTS
054	68	NOP	
055	76	LBL	PRINT M's, K's
056	15	E	
057	86	STF	Do not R/S
058	00	00	in loop E'
059	01	1	
060	01	1	
061	42	STO	Counter
062	00	00	
063	01	1	
064	09	9	
065	42	STO	Pointer
066	01	01	
067	10	E'	Output: M's, K's
068	81	RST	
069	76	LBL	GUY DATA
070	12	B	
071	01	1	
072	42	STO	Counter
073	00	00	
074	05	5	
075	42	STO	Pointer
076	01	01	
077	10	E'	Input: L
078	02	2	
079	42	STO	Counter
080	00	00	
081	01	1	
082	03	3	
083	42	STO	Pointer
084	01	01	
085	10	E'	Input: K_c, e
086	43	RCL	
087	14	14	e
088	71	SBR	Compute
089	40	IND	Ψ(e)'s
090	30	30	
091	02	2	
092	06	6	
093	42	STO	Pointer
094	00	00	
095	43	RCL	K_c
096	13	13	
097	71	SBR	Compute
098	04	04	$k_c\Psi_i(e)\Psi_j(e)$
099	41	41	
100	81	RST	
101	42	STO	SBR CANTI. Ψ's
102	02	02	Z
103	55	÷	
104	43	RCL	L
105	05	05	
106	95	=	
107	42	STO	$= \Psi_y$

LOC.	CODE	KEY	COMMENTS
108	00	00	
109	75	-	
110	33	X^2	
111	95	=	
112	49	PRD	$= \Psi_x$
113	02	02	
114	43	RCL	Z/L
115	00	00	
116	33	X^2	
117	65	×	
118	03	3	
119	75	-	
120	02	2	
121	65	×	
122	43	RCL	Z/L
123	00	00	
124	45	Y^x	
125	03	3	
126	95	=	
127	42	STO	$= \Psi_v$
128	01	01	
129	06	6	
130	55	÷	
131	43	RCL	L
132	05	05	
133	33	X^2	
134	75	-	
135	24	CE	
136	65	×	
137	02	2	
138	65	×	
139	43	RCL	Z/L
140	00	00	
141	95	=	
142	42	STO	$= \Psi_v''$
143	03	03	
144	02	2	
145	55	÷	
146	43	RCL	L
147	05	05	
148	75	-	
149	24	CE	
150	65	×	
151	03	3	
152	65	×	
153	43	RCL	Z/L
154	00	00	
155	95	=	
156	42	STO	$= \Psi_x''$
157	04	04	
158	92	RTN	
159	42	STO	SBR BALL. Ψ's
160	02	02	Z

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Program Title: INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES — 2.13

LOC.	CODE	KEY	COMMENTS
161	55	-	
162	43	RCL	L
163	05	05	
164	95	=	
165	42	STO	$=\Psi_\phi$
166	00	00	
167	94	+/-	
168	65	×	
169	03	3	
170	55	÷	
171	43	RCL	L
172	05	05	
173	95	=	
174	42	STO	$=\Psi_x''$
175	04	04	
176	55	÷	
177	43	RCL	L
178	05	05	
179	95	=	
180	42	STO	$=\Psi_v''$
181	03	03	
182	43	RCL	Z
183	02	02	
184	55	÷	
185	02	2	
186	75	-	
187	24	CE	
188	65	×	
189	43	RCL	Z/L
190	00	00	
191	33	X^2	
192	95	=	
193	42	STO	$=\Psi_x$
194	02	02	
195	03	3	
196	75	-	
197	43	RCL	Z/L
198	00	00	
199	33	X^2	
200	95	=	
201	65	×	
202	43	RCL	Z/L
203	00	00	
204	55	÷	
205	02	2	
206	95	=	
207	42	STO	$=\Psi_v$
208	01	01	
209	92	RTN	
210	76	LBL	BALL JOINT BASE
211	16	A'	
212	01	1	
213	05	5	
214	09	9	SER Ψ address
215	61	GTO	
216	00	00	
217	47	47	
218	76	LBL	NON-UNIFORM TOWER
219	18	C'	
220	03	3	Request for EI, GJ, m, Im at each Z
221	07	7	
222	07	7	
223	61	GTO	
224	02	02	
225	31	31	
226	76	LBL	UNIFORM TOWER
227	13	C	
228	03	3	Input EI, GJ, m, Im only once...
229	09	9	
230	03	3	
231	42	STO	SER address
232	17	18	
233	03	3	
234	42	STO	Counter
235	00	00	
236	05	5	
237	42	STO	Pointer
238	01	01	
239	10	E'	Input: L, L', N
240	43	RCL	
241	06	06	L'
242	55	÷	
243	43	RCL	N
244	07	07	
245	95	=	
246	42	STO	ΔZ
247	31	31	
248	55	÷	
249	03	3	
250	95	=	
251	42	STO	ΔZ/3
252	32	32	
253	65	×	
254	04	4	
255	95	=	
256	42	STO	4ΔZ/3
257	33	33	
258	71	SBR	Output: Z
259	03	03	Input: EI, GJ, m, Im
260	77	77	
261	02	2	
262	49	PRD	2ΔZ/3
263	32	32	
264	69	OP	
265	37	37	
266	43	RCL	2ΔZ/3
267	32	32	
268	48	EXC	
269	33	33	4ΔZ/3
270	42	STO	2ΔZ/3
271	32	32	
272	43	RCL	ΔZ — Simpson's integration loop
273	31	31	
274	44	SUM	Z
275	08	08	
276	71	SBR	Evaluate integrand
277	40	IND	
278	17	18	
279	97	DSZ	
280	07	07	
281	02	02	
282	66	66	
283	43	RCL	ΔZ
284	31	31	
285	44	SUM	Z
286	08	08	
287	04	4	
288	22	INV	
289	49	PRD	ΔZ/3
290	32	32	
291	71	SBR	Evaluate integrand for last interval
292	40	IND	
293	17	18	
294	43	RCL	
295	05	05	L
296	32	X:T	
297	43	RCL	Z
298	08	08	
299	67	EQ	
300	03	03	Z=L ?
301	03	03	
302	81	RST	
303	43	RCL	
304	05	05	Z
305	33	X^2	
306	22	INV	
307	49	PRD	$K_{\phi\phi}$
308	28	28	
309	43	RCL	
310	26	26	K_{vv}
311	75	-	
312	43	RCL	
313	27	27	K_{vx}
314	33	X^2	
315	55	÷	
316	43	RCL	
317	29	29	K_{xx}
318	95	=	
319	42	STO	
320	25	25	k_{vv}
321	43	RCL	
		(21)	

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Program Listing

Program Title INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES P 14

LOC.	CODE	KEY	COMMENTS
322	31	21	m_{vv}
323	85	+	
324	43	RCL	K_{vx}
325	27	27	
326	55	÷	
327	43	RCL	K_{xx}
328	29	29	
329	65	×	
330	53	(	
331	24	CE	
332	65	×	
333	43	RCL	m_{xx}
334	24	24	
335	75	-	
336	02	2	
337	65	×	
338	43	RCL	M_{vx}
339	22	22	
340	54	)	
341	95	=	
342	42	STO	m_{uu}
343	19	19	
344	81	RST	
345	76	LBL	ROTOR & NACELLE DATA
346	14	D	
347	04	4	
348	42	STO	Counter
349	00	00	
350	01	1	
351	05	5	
352	42	STO	Pointer
353	01	01	
354	10	E'	Input: I_{xx}, I_{zz},
355	43	RCL	m_{vv}, m_{uu}, q
356	17	17	m_{vv}
357	44	SUM	
358	19	19	M_{uu}
359	44	SUM	M_{vv}
360	21	21	
361	65	×	
362	43	RCL	q
363	18	18	
364	95	=	
365	94	+/-	
366	44	SUM	$M_{u\phi}$
367	20	20	
368	43	RCL	I_{xx}
369	15	15	
370	44	SUM	M_{xx}
371	24	24	
372	43	RCL	I_{zz}
373	16	16	
374	44	SUM	$M_{\phi\phi}$
375	23	23	

LOC.	CODE	KEY	COMMENTS
376	81	RST	
377	86	STF	SBR NON-UNIF TOWER
378	00	00	
379	01	1	
380	42	STO	Counter
381	00	00	
382	08	8	
383	42	STO	pointer
384	01	01	Output: z
385	10	E'	
386	22	INV	
387	86	STF	
388	00	00	
389	04	4	
390	42	STO	Counter
391	00	00	
392	10	E'	Input: EI, GJ,
393	43	RCL	m, I_m
394	08	08	z SBR UNIF. TOWER
395	71	SBR	Compute
396	40	IND	Ψ_{ij}, Ψ''_{ij}
397	30	30	
398	43	RCL	$\Delta z/3$; $2\Delta z/3$
399	32	32	
400	65	×	
401	43	RCL	I_m
402	12	12	
403	65	×	
404	43	RCL	Ψ_ϕ
405	00	00	
406	33	X^2	
407	95	=	
408	44	SUM	$m_{\phi\phi}$
409	23	23	
410	43	RCL	$\Delta z/3$; $2\Delta z/3$
411	32	32	
412	65	×	
413	43	RCL	GJ
414	10	10	
415	95	=	
416	44	SUM	$K_{\phi\phi}$
417	28	28	
418	02	2	
419	01	1	
420	42	STO	Pointer
421	00	00	
422	43	RCL	$\Delta z/3$; $2\Delta z/3$
423	32	32	
424	65	×	
425	43	RCL	m
426	11	11	
427	95	=	
428	71	SBR	Compute
429	04	04	$m(z)\Psi(z)\Psi(z)$

LOC.	CODE	KEY	COMMENTS
430	41	41	
431	02	2	
432	06	6	
433	42	STO	Pointer
434	00	00	
435	43	RCL	$\Delta z/3$; $2\Delta z/3$
436	32	32	
437	65	×	
438	43	RCL	EI
439	09	09	
440	95	=	
441	42	STO	SBR INTEGRAND
442	34	34	Integrand
443	65	×	
444	43	RCL	Ψ_v
445	01	01	
446	33	X^2	
447	95	=	
448	74	SM*	K_{vv} ; m_{vv}
449	00	00	
450	69	OP	Move pointer
451	20	20	
452	43	RCL	Integrand
453	34	34	
454	65	×	
455	43	RCL	Ψ_v''
456	03	03	
457	48	EXC	Ψ_v
458	01	01	
459	65	×	
460	43	RCL	Ψ_x
461	02	02	
462	95	=	
463	74	SM*	K_{vx} ; M_{vx}
464	00	00	
465	02	2	
466	44	SUM	Move pointer
467	00	00	
468	43	RCL	Integrand
469	34	34	
470	65	×	
471	43	RCL	Ψ_x''
472	04	04	
473	48	EXC	Ψ_x
474	02	02	
475	33	X^2	
476	95	=	
477	74	SM+	K_{xx} ; M_{xx}
478	00	00	
479	92	RTN	

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Program Record 102

TI Programmable 59

"VIBES"

VIBRATIONAL FREQUENCIES

OF WIND TURBINE SYSTEMS

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Program Abstract

Program Title VIBRATIONAL FREQUENCIES OF WIND TURBINE SYSTEMS | P.1

evaluates the free vibration frequencies for a wind turbine system described by the equations given below. The program may also be used for computing rotor rotation speeds as a function of frequency.

$$\begin{bmatrix} M_{uu} & 0 & M_{u\phi} & 0 \\ 0 & M_{vv} & 0 & M_{v\chi} \\ M_{u\phi} & 0 & M_{\phi\phi} & 0 \\ 0 & M_{v\chi} & 0 & M_{\chi\chi} \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{\phi} \\ \ddot{\chi} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega I_r \\ 0 & 0 & -\Omega I_r & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{\phi} \\ \dot{\chi} \end{Bmatrix} + \begin{bmatrix} k_{uu} & 0 & 0 & 0 \\ 0 & k_{vv} & 0 & k_{v\chi} \\ 0 & 0 & k_{\phi\phi} & 0 \\ 0 & k_{v\chi} & 0 & k_{\chi\chi} \end{bmatrix} \begin{Bmatrix} u \\ v \\ \phi \\ \chi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

To run the program proceed as follows:

1. LOAD CARD SIDES 1, 2, AND 3.

2. PRINT HEADING & INPUT MASS MOMENT OF INERTIA OF ROTOR ABOUT IT'S SPIN AXIS.
   Press A ; Input: ROT I (I_r).

3. INPUT INERTIA AND STIFFNESS COEFFICIENTS.*
   Press B; Input: M_{uu}, $M_{u\phi}$, M_{vv}, $M_{v\chi}$, $M_{\phi\phi}$, $M_{\chi\chi}$, k_{uu}, k_{vv}, $k_{v\chi}$, $k_{\phi\phi}$, $k_{\chi\chi}$

4. COMPUTE FREQUENCIES CORRESPONDING TO INFINITE AND ZERO ROTOR SPEEDS.**
   Press C ; Output: $\omega_{\infty 1}$, $\omega_{\infty 2}$, $\omega_{\infty 3}$, $\omega_{\infty 4}$ & $\omega_{u\phi 1}$, $\omega_{u\phi 2}$, $\omega_{v\chi 1}$, $\omega_{v\chi 2}$

5. OBTAIN DATA POINTS FOR PLOTTING FREQUENCIES, ω_{oi} VS. ROTOR SPEED Ω.***
   Press D ; Input: ω ; Output: RTW (Ω)
   Input: ω ; Output: RTW (Ω)
   Repeat untill sufficient data points have been obtained.

---

* Program 101 — INERTIA AND STIFFNESS COEFFICIENTS OF WIND TURBINES — can be used to evaluate the coefficients Mij and kij for the wind turbine system. Both programs 101 and 102 use the same data registers for these coefficients so that step 3 above can be skipped if the two programs are run in sequence and provided the contents of bank 4 are not destroyed (do not clear memories, do not Load card side 4, do not turn off the calculator before running Program 102)

** A flashing display indicates an error condition (probably an erroneous coefficient input resulting in complex frequencies.

*** This step must be performed after step 4. Display will flash for unacceptable frequency input, e.g. $\omega = 0$; $\omega_{o1} < \omega < \omega_{o2}$, $\omega_{\infty 2} < \omega < \omega_{o3}$, $\omega_{\infty 3} < \omega < \omega_{\infty 4}$.

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User Instructions

Program Title VIBRATIONAL FREQUENCIES OF WIND TURBINE SYSTEMS — P. 2

STEP	PROCEDURE	PRINTOUT PC-100A	DISPLAY
1	LOAD CARD SIDES 1, 2, AND 3.		
2	PRINT HEADING & INPUT ROTOR INERTIA.		
2a	Press A.	SYSTEM NATURAL FREQ.	
		ROT I	0.
2b	Enter I_r, mass moment of rotor about its spin axis.		iiii
2c	Press R/S.	iiii	iiii
3	INPUT INERTIA AND STIFFNESS COEFFS. See note 1.		
3a	Press B.	Muu	0.
3b	Enter M_{uu}.	uu.uu	uu.uu
3c	Press R/S.		
3d	Enter $M_{u\phi}$.	MuQ	0.
3e	Press R/S.	uφ.uφ	uφ.uφ
3f	Enter M_{vv}	M VV	0.
3g	Press R/S	vv.vv	vv.vv
3h	Enter M_{vx}	M VX	0.
3i	Press R/S	vx.vx	vx.vx
3j	Enter $M_{\phi\phi}$	M QQ	0.
3k	Press R/S	φφ.φφ	φφ.φφ
3l	Enter M_{xx}	M XX	0.
3m	Press R/S	xx.xx	xx.xx
3n	Enter k_{uu}	K UU	0.
3o	Press R/S	kk.uu	kk.uu
3p	Enter k_{vv}	K VV	0.
3q	Press R/S	kk.vv	kk.vv
3r	Enter k_{vx}	K VX	0.
3s	Press R/S	kk.vx	kk.vx
3t	Enter $k_{\phi\phi}$	K QQ	0.
3u	Press R/S	kk.φφ	kk.φφ
3v	Enter k_{xx}	K XX	0.
3w	Press R/S	kk.xx	kk.xx

Note 1: This step may be skipped if the coefficients M_{ij} and k_{ij} are computed by Program 101 (See foot note * on page 1)

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Program Title VIBRATIONAL FREQUENCIES OF WIND TURBINE SYSTEMS | P 3

STEP	PROCEDURE	PRINTOUT PC-100A	DISPLAY
4	COMPUTE FREQUENCIES AT INFINITE AND ZERO ROTOR SPEEDS.		
4a	Press C.	INFINTE ROTOR SPEED	
		0.	
	See foot note ** on page 1 for error conditions.	ω.ωω2	
		ω.ωω3	
		INFINITY	
		ZERO ROTOR SPEED	
		ω.φω1 Q-U	
		ω.φω2 Q-U	
		ω.xv1 X-V	
		ω.xv2 X-V	
			ω.xv2
5	OBTAIN DATA POINTS FOR PLOTTING NATURAL FREQUENCIES VS. ROTOR SPEED.		
5a	Press D.	W ?	
5b	Enter ω.		ω.ωω
5c	Press R/S.	ω.ωω	
		Ω.ΩΩ RT W	
		W?	
5d	Enter ω.		ω.ωω
5e	Press R/S.	ω.ωω	
		Ω.ΩΩ RT W	
	Repeat until enough data points have been obtained.		
	See foot note *** on page 1 for unacceptable values of ω.		

RUNNING THE PROGRAM WITHOUT A PC-100A PRINTER

To run this program without a printer, all the print commands (PRT $\begin{smallmatrix}CP\\06\end{smallmatrix}$) must be replaced by a R/S. This modification will cause the program to halt and display output values, the user notes the output and presses R/S to resume execution.

To modify the program load card sides 1, 2, and 3 and press the following key sequence GTO [X] [X] [X] LRN R/S LRN with [X] [X] [X] = 096, 104, 112, 131, 295, 304, and 396.

The program will display 2437450000 for $\omega_{004} = 00$.

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User Information

Program Title VIBES | P 4

USER DEFINED KEYS

A - Initialize and input ROT I (I_r)
A'- ~
B - Input M_{ij} and K_{ij}
B'- ~
C - Compute $\omega_{\omega i}$, $\omega_{\theta i}$
C'- ~
D - Input ω, compute ROT ω (Ω)
D'- ~
E - Used (quadratic eqn.)
E'- ~

☐ t Register
☑ Absolute Addresses
☑ PC - 100A*
☑ Disturbs Pending Operations

Partition 479 , 59 **
Program Steps 399
Library Module —
Card Sides 3
Angular Mode —
Parentheses Level 2
SBR Levels 1

* See bottom page 3.
** Initial partition for TI 59

Flags	0	1	2	3	4
	5	6	7	8	9

* DATA REGISTERS *

Contents	Reg.	Comments	Contents	Reg.	Comments
counter; A	00	Computed	3645363717. "SYSTE"	30	Label
pointer; B	01	"	3000311337. "M NAT"	31	"
pointer; C	02	"	4135132700. "URAL"	32	"
ω^2	03	Input	2135173440. "FREQ."	33	"
—	04	—	2431212431. "INFIN"	34	"
—	05	—	2437170035. "ITE R"	35	"
—	06	—	3237323500. "OTOR "	36	"
—	07	—	3633171716. "SPEED"	37	"
—	08	—	46. " Z"	38	"
—	09	—	1735320035. "ERO R"	39	"
—	10	—	430071. " W?"	40	"
—	11	—	35370043. "RT W"	41	"
—	12	—	$A_{u\phi}$	42	Computed
—	13	—	$B_{u\phi}$	43	"
—	14	—	$C_{u\phi}$	44	"
—	15	—	A_{vx}	45	"
—	16	—	B_{vx}	46	"
—	17	—	C_{vx}	47	"
I_r (ROT I)	18	Input	3532370024. "ROT I"	48	Label
M_{uu}	19	"	30004141. " MUU"	49	"
$M_{u\phi}$	20	"	30004134. " MUQ"	50	"
M_{vv}	21	"	30004242. " MVV"	51	"
M_{vx}	22	"	30004244. " MVX"	52	"
$M_{\phi\phi}$	23	"	30003434. " MQQ"	53	"
M_{xx}	24	"	30004444. " MXX"	54	"
K_{uu}	25	"	26004141. " KUU"	55	"
K_{vv}	26	"	26004242. " KVV"	56	"
K_{vx}	27	"	26004244. " KVX"	57	"
$K_{\phi\phi}$	28	"	26003434. " KQQ"	58	"
K_{xx}	29	"	26004444. " KXX"	59	"

Notes:

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Program Title VIBES | P.5

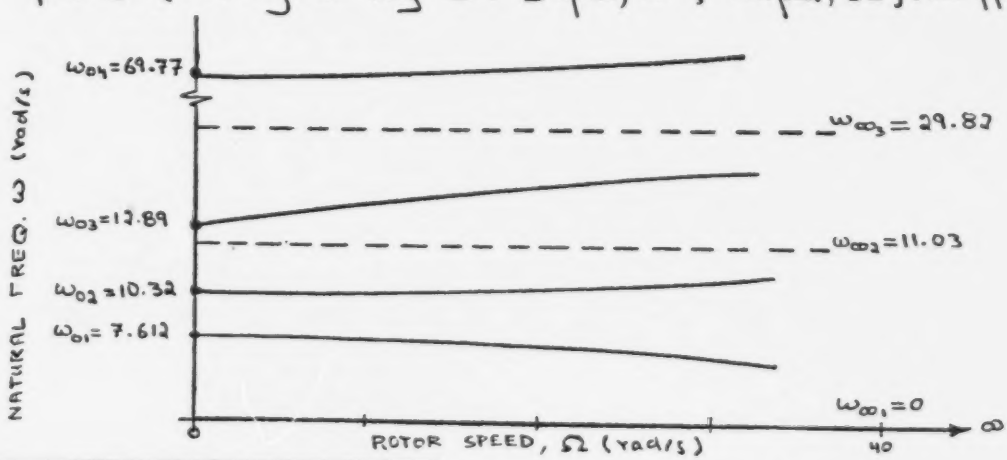
Find the system natural frequencies at zero rotor speed and the frequencies corresponding to infinite rotor speed for a wind turbine system described by the equations of motion given on page 1. The mass moment of inertia of the rotor about its spin axis is, $I_r = 570$ lb-s^2-ft. The inertia coefficients, M_{ij} and the stiffness coefficients K_{ij} are as follows*:

M_{uu} = 56.43326281	lb-s^2/ft	K_{uu} = 6865.538931	lb/ft
$M_{u\phi}$ = -83.85	lb-s^2	K_{vv} = 51949.54887	lb/ft
M_{vv} = 58.40760828	lb-s^2/ft	K_{vx} = 375650.9686	lb
M_{vx} = 18.21937687	lb-s^2	$K_{\phi\phi}$ = 51437.5	lb-ft
$M_{\phi\phi}$ = 774.3475923	lb-s^2-ft	K_{xx} = 3133349.291	lb-ft
M_{xx} = 716.5607592	lb-s^2-ft		

See the printout on page 6 for results.

The frequencies, ω_{oi} can now be plotted against rotor speed as outlined below:

1. There will be four curves; each will start at a value, ω_{oi}, and then as the rotor speed increases, the frequency curves asymptotically approach a $\omega_{\infty i}$
2. Arrange the natural frequencies $\omega_{\phi u1}$, $\omega_{\phi u2}$, ω_{xv1}, and ω_{xv2} in ascending order and plot them on the vertical axis.
3. Draw the asymptotes $\omega_{\infty 1}$, $\omega_{\infty 2}$, and $\omega_{\infty 3}$.
4. Sketch in the frequency curves.
5. Find the exact shape of the curves by obtaining additional data points (User defined key **D**: Input, ω; Output, Ω) (see pp. 7 and 8.)



* The coefficients M_{ij}, K_{ij} were obtained from the sample problem in program 101.

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Sample Run

Program Title VIBES — P.6

ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
	Load card sides 1, 2, and 3 (see note 1)				
	A	SYSTEM NATURAL FREQ. ROT I 570.	INITIALLIZE AND INPUT ROT I (I_r)	0. 570.	
	B	M UU	INPUT M_{ij} AND k_{ij}	0.	see note 2
56.433...	R/S	56.43326281 M UQ		0.	
83.85	+/- R/S	-83.85 M VV		0.	
58.407...	R/S	58.40760828 M VX		0.	
18.219...	R/S	18.21937687 M QQ		0.	
774.34...	R/S	774.3475923 M XX		0.	
716.56...	R/S	716.5607592 K UU		0.	
6865.5...	R/S	6865.538931 K VV		0.	
51949.5...	R/S	51949.54887 K VX		0.	
375850...	R/S	375850.9686 K QQ		0.	
51437.5	R/S	51437.5 K XX		0.	
3133349...	R/S	3133349.291		3133349.291	
	C	INFINITE ROTOR SPEED	COMPUTE FREQUENCIES		
		0.	$= \omega_{\infty 1}$		
		11.02985331	$= \omega_{\infty 2}$		
		29.82333307	$= \omega_{\infty 3}$		
		INFINITY	$= \omega_{\infty 4}$		
		ZERO ROTOR SPEED			
		7.612297423 Q-U	$= \omega_{01}$		
		12.89190393 Q-U	$= \omega_{03}$		
		10.31656562 X-V	$= \omega_{02}$		
		69.77092522 X-V	$= \omega_{03}$	69.77092522	

Note 1: There is no need to load card side 4 (See footnote * on page 1)

Note 2: If these coefficients are computed by Program 101 (see footnote * on page 1) then these display quantities would be the computed quantities and not zero

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Sample Run

Program Title VIBES — F 7

ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
			COMPUTE DATA POINTS FOR ω_{01} VS Ω CURVE (OBTAIN Ω AS A FUNCTION OF ω)		
	D				
7.6	R/S	W ? 7.6 1.483163761 RT W	= Frequency = Rotor speed (Ω)	430071	" ω ? "
7	R/S	W ? 7. 11.74272387 RT W		430071	" ω ? "
6.5	R/S	W ? 6.5 17.36152832 RT W		"	"
6	R/S	W ? 6. 22.92042849 RT W		"	"
5.5	R/S	W ? 5.5 28.817558 RT W		"	"
			COMPUTE DATA POINTS FOR ω_{02} VS. Ω CURVE		
10.32	R/S	W ? 10.32 1.049039004 RT W		430071	" ω ? "
10.4	R/S	W ? 10.4 5.466866848 RT W		"	"
10.5	R/S	W ? 10.5 8.78223466 RT W		"	"
10.6	R/S	W ? 10.6 12.02706516 RT W		"	"
10.7	R/S	W ? 10.7 15.82364726 RT W		"	"
10.8	R/S	W ? 10.8 21.05940258 RT W		"	"
10.9	R/S	W ? 10.9 30.4071521 RT W		"	"

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Sample Run

Program Title VIBES

P 5

ENTER	PRESS	PRINTOUT	COMMENTS	DISPLAY	COMMENTS
			COMPUTE DATA POINTS FOR ω_{04} VS. Ω CURVE		
		W ?		430071	"ω?"
12.9	R/S	12.9			
		1.298564755 RT W			
		W ?		〃	〃
13	R/S	13.			
		4.74751613 RT W			
		W ?		〃	〃
14	R/S	14.			
		15.56331139 RT W			
		W ?		〃	〃
15	R/S	15.			
		22.3068585 RT W			
		W ?		〃	〃
16	R/S	16.			
		28.29521429 RT W			
		W ?		〃	〃
17	R/S	17.			
		34.07737257 RT W			
			COMPUTE DATA POINTS FOR ω_{04} VS. Ω CURVE		
		W ?		430071	"ω?"
69.8	R/S	69.8			
		2.599480238 RT W			
		W ?		〃	
70	R/S	70.			
		7.298034628 RT W			
		W ?		〃	
71	R/S	71.			
		16.923014 RT W			
		W ?		〃	
72	R/S	72.			
		22.81725059 RT W			
		W ?		〃	
73	R/S	73.			
		27.49731945 RT W			
		W ?		〃	
74	R/S	74.			
		31.51078338 RT W	END OF RUN		

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Program Title VIBES | pg

The linear, second order differential equations of a wind turbine system given on page 1 are known to have a solution of the form $\{X(t)\} = \{X(0)\} e^{i\omega t}$ where $\{X(t)\}$ is the vector of translational and rotational displacements at the upper end of the tower, $[U \; V \; \Phi \; X]^T$. Substitution of this solution into the equations of motion yields the following set of equations:

$$\begin{bmatrix} (k_{uu} - M_{uu}\omega^2) & 0 & (-M_{u\phi}\omega^2) & 0 \\ 0 & (k_{vv} - M_{w}\omega^2) & 0 & (k_{vx} - M_{vx}\omega^2) \\ (-M_{u\phi}\omega^2) & 0 & (k_{\phi\phi} - M_{\phi\phi}\omega^2) & (i I_r \Omega \omega) \\ 0 & (k_{vx} - M_{vx}\omega^2) & (-i I_r \Omega \omega) & (k_{xx} - M_{xx}\omega^2) \end{bmatrix} \begin{Bmatrix} U_o \\ V_o \\ \phi_o \\ X_o \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Equating the determinant of the square matrix to zero (nontrivial solution) and solving the resulting polynomial for the rotor spin rate, Ω (RT ω) yields

$$\Omega^2 = \frac{G_{u\phi}(\omega^2)\, H_{vx}(\omega^2)}{I_r^2\, \omega^2 (k_{uu} - M_{uu}\omega^2)(k_{vv} - M_{vv}\omega^2)} \quad \cdots (A)$$

where $G_{u\phi}(\omega^2)$ and $H_{vx}(\omega^2)$ are quadratic polynomials in ω^2

$$G_{u\phi}(\omega^2) = k_{uu}k_{\phi\phi} - (k_{uu}M_{\phi\phi} + k_{\phi\phi}M_{uu})(\omega^2) + (M_{uu}M_{\phi\phi} - M_{u\phi}^2)(\omega^2)^2$$

$$H_{vx}(\omega^2) = k_{xx}k_{vv} - k_{vx}^2 - (k_{xx}M_{vv} + k_{vv}M_{xx} - 2k_{vx}M_{vx})(\omega^2) + (M_w M_{xx} - M_{vx}^2)(\omega^2)^2$$

Now if the rotor spin rate, Ω, is zero then the four system natural frequencies are the roots of the quadratic polynomials $G_{u\phi}$ and H_{vx}. On the other hand, the four frequencies which give infinite rotor speed (i.e. make the denominator of equation (A) zero) are

$$\omega_{oi1} = 0 \;\; ; \;\; \omega_{oi2} = (k_{uu}/M_{uu})^{1/2} \;\; ; \;\; \omega_{oi3} = (k_{vv}/M_{vv})^{1/2} \;\; ; \;\; \omega_{oi4} = \infty$$

Data points for the ω_{oi} vs. Ω curves are obtained by using equation (A) to compute the rotor speed, Ω as a function of frequency, ω.

Reference:
Thresher, et al., Modeling the Response of Wind Turbines to Atmospheric Turbulance, Oregon State University Report No. RLO/2227-81/2 (1981).

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Program Listing

Program Title VIBES — P 10

LOC.	CODE	KEY	COMMENTS
000	76	LBL	ENTER ROT I
001	11	A	
002	43	RCL	"SYSTE "
003	30	30	
004	69	OP	
005	01	01	
006	43	RCL	"M NAT "
007	31	31	
008	69	OP	
009	02	02	
010	43	RCL	"URAL "
011	32	32	
012	69	OP	
013	03	03	
014	43	RCL	"FRED."
015	33	33	
016	69	OP	
017	04	04	
018	69	OP	Print heading
019	05	05	
020	69	OP	
021	00	00	
022	98	ADV	
023	01	1	
024	42	STO	Loop counter
025	00	00	
026	01	1	
027	08	8	
028	42	STO	Variable pointer
029	01	01	
030	04	4	
031	08	8	
032	42	STO	Label pointer
033	02	02	
034	73	RC*	Label
035	02	02	
036	69	OP	
037	02	02	
038	69	OP	Print fairi
039	05	05	
040	73	RC*	Display variable
041	01	01	
042	91	R/S	Print entry
043	99	PRT	
044	72	ST*	Store entry
045	01	01	
046	69	OP	
047	21	21	Indirect reading Loop
048	69	OP	
049	22	22	
050	97	DSZ	Move pointer
051	00	00	
052	00	00	
053	34	34	
054	98	ADV	
055	91	R/S	
056	76	LBL	ENTER M's, k's
057	12	B	
058	01	1	
059	01	1	
060	42	STO	
061	00	00	
062	01	1	
063	09	9	
064	42	STO	
065	01	01	
066	04	4	
067	09	9	
068	61	GTO	Read M's & k's
069	00	00	
070	33	32	
071	76	LBL	COMPUTE ω's
072	13	C	
073	98	ADV	
074	43	RCL	"INFIN"
075	34	34	
076	69	OP	
077	01	01	
078	43	RCL	"ITE R "
079	35	35	
080	69	OP	
081	02	02	
082	43	RCL	"OTOR "
083	36	36	
084	69	OP	
085	03	03	
086	43	RCL	"SPEED"
087	37	37	
088	69	OP	
089	04	04	
090	69	OP	Print heading
091	05	05	
092	69	OP	
093	00	00	
094	98	ADV	
095	25	CLR	0.
096	99	PRT	$=\omega_{001}$
097	43	RCL	K_{uu}
098	25	25	
099	55	÷	
100	43	RCL	M_{uu}
101	19	19	
102	95	=	
103	34	FX	
104	99	PRT	$=\omega_{002}$
105	43	RCL	K_{vv}
106	26	26	
107	55	-	
108	43	RCL	M_{vv}
109	21	21	
110	95	=	
111	34	FX	
112	99	PRT	$=\omega_{003}$
113	43	RCL	"INFIN"
114	34	34	
115	69	OP	
116	02	02	
117	02	2	
118	04	4	
119	03	3	
120	07	7	
121	04	4	
122	05	5	
123	00	0	
124	00	0	
125	00	0	
126	00	0	
127	69	OP	
128	03	03	
129	69	OP	"INFINITY"
130	05	05	$=\omega_{004}$
131	98	ADV	
132	98	ADV	
133	43	RCL	" Z"
134	38	38	
135	69	OP	
136	01	01	
137	43	RCL	"ERO R"
138	39	39	
139	69	OP	
140	02	02	
141	43	RCL	"OTOR "
142	36	36	
143	69	OP	
144	03	03	
145	43	RCL	"SPEED"
146	37	37	
147	69	OP	
148	04	04	
149	69	OP	Print heading
150	05	05	
151	69	OP	
152	00	00	
153	98	ADV	
154	43	RCL	M_{uu}
155	19	19	
156	65	×	
157	43	RCL	$M_{\phi\phi}$
158	23	23	
159	75	-	
160	43	RCL	$M_{u\phi}$

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Program Listing

Program Title VIBES — P11

LOC.	CODE	KEY	COMMENTS
161	20	20	
162	33	X^2	
163	95	=	
164	42	STO	$= A_{u\phi}$
165	42	42	
166	42	STO	= A
167	00	00	
168	43	RCL	k_{uu}
169	25	25	
170	65	×	
171	43	RCL	$M_{\phi\phi}$
172	23	23	
173	85	+	
174	43	RCL	$k_{\phi\phi}$
175	28	28	
176	65	×	
177	43	RCL	M_{uu}
178	19	19	
179	95	=	
180	94	+/-	
181	42	STO	$= B_{u\phi}$
182	43	43	
183	42	STO	= B
184	01	01	
185	43	RCL	k_{uu}
186	25	25	
187	65	×	
188	43	RCL	$k_{\phi\phi}$
189	28	28	
190	95	=	
191	42	STO	$= C_{u\phi}$
192	44	44	
193	42	STO	= C
194	02	02	
195	03	3	
196	04	4	
197	02	2	
198	00	0	
199	04	4	
200	01	1	
201	69	DP	"u-Φ"
202	04	04	
203	71	SBR	Solve quadratic eq
204	15	E	
205	43	RCL	M_{vv}
206	21	21	
207	65	×	
208	43	RCL	M_{xx}
209	24	24	
210	75	-	
211	43	RCL	M_{vx}
212	22	22	
213	33	X^2	
214	95	=	
215	42	STO	$= A_{vx}$
216	45	45	
217	42	STO	= A
218	00	00	
219	43	RCL	k_{xx}
220	29	29	
221	65	×	
222	43	RCL	M_{vv}
223	21	21	
224	85	+	
225	43	RCL	k_{vv}
226	26	26	
227	65	×	
228	43	RCL	M_{xx}
229	24	24	
230	75	-	
231	02	2	
232	65	×	
233	43	RCL	k_{vx}
234	27	27	
235	65	×	
236	43	RCL	M_{vx}
237	22	22	
238	95	=	
239	94	+/-	
240	42	STO	$= B_{vx}$
241	46	46	
242	42	STO	= B
243	01	01	
244	43	RCL	k_{xx}
245	29	29	
246	65	×	
247	43	RCL	k_{vv}
248	26	26	
249	75	-	
250	43	RCL	k_{vx}
251	27	27	
252	33	X^2	
253	95	=	
254	42	STO	$= C_{u\phi}$
255	47	47	
256	42	STO	= C
257	02	02	
258	04	4	
259	04	4	
260	02	2	
261	00	0	
262	04	4	
263	02	2	
264	69	DP	"v-x"
265	04	04	
266	76	LBL	QUADRATIC EQ. SBR.
267	15	E	
268	43	RCL	
269	01	01	
270	55	÷	
271	02	2	
272	55	÷	
273	43	RCL	
274	00	00	
275	95	=	
276	42	STO	
277	01	01	
278	33	X^2	
279	75	-	
280	43	RCL	
281	02	02	
282	55	÷	
283	43	RCL	
284	00	00	
285	95	=	
286	34	fX	
287	42	STO	
288	02	02	
289	85	+	
290	43	RCL	
291	01	01	
292	95	=	
293	94	+/-	
294	34	fX	
295	69	DP	First root
296	06	06	
297	43	RCL	
298	02	02	
299	75	-	
300	43	RCL	
301	01	01	
302	95	=	
303	34	fX	
304	69	DP	Second root
305	06	06	
306	98	ADV	
307	92	RTN	
308	76	LBL	COMPUTE ROTW
309	14	D	
310	69	DP	
311	00	00	
312	98	ADV	
313	43	RCL	"w?"
314	40	40	
315	69	DP	
316	02	02	
317	69	DP	
318	05	05	
319	91	R/S	
320	99	PPT	
321	33	X^2	

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Program Listing

Program Title VIBES — P 12

LOC.	CODE	KEY	COMMENTS
322	42	STO	$= \omega^2$
323	03	03	
324	43	RCL	"RT ω"
325	41	41	
326	69	OP	
327	04	04	
328	53	(	
329	43	RCL	ω^2
330	03	03	
331	33	X^2	
332	65	×	
333	43	RCL	$A_{u\phi}$
334	42	42	
335	85	+	
336	43	RCL	ω^2
337	03	03	
338	65	×	
339	43	RCL	$B_{u\phi}$
340	43	43	
341	85	+	
342	43	RCL	$C_{u\phi}$
343	44	44	
344	54	)	$= G_{u\phi}$
345	65	×	
346	53	(	
347	43	RCL	ω^2
348	03	03	
349	33	X^2	
350	65	×	
351	43	RCL	$A_{v\pi}$
352	45	45	
353	85	+	
354	43	RCL	ω^2
355	03	03	
356	65	×	
357	43	RCL	$B_{v\pi}$
358	46	46	
359	85	+	
360	43	RCL	$C_{v\pi}$
361	47	47	
362	54	)	$= H_{v\pi}$
363	55	÷	
364	53	(	
365	53	(	
366	43	RCL	k_{uu}
367	25	25	
368	75	-	
369	43	RCL	M_{uu}
370	19	19	
371	65	×	
372	43	RCL	ω^2
373	03	03	
374	54	)	
375	65	×	
376	53	(	
377	43	RCL	k_{vv}
378	26	26	
379	75	-	
380	43	RCL	M_{vv}
381	21	21	
382	65	×	
383	43	RCL	ω^2
384	03	03	
385	54	)	
386	65	×	
387	43	RCL	ω^2
388	03	03	
389	65	×	
390	43	RCL	ROT I I_v
391	18	18	
392	33	X^2	
393	54	)	
394	95	=	
395	34	FX	= RT ω Ω
396	69	OP	
397	06	06	
398	14	D	

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Appendix E: Program MITGUY

Program MITGUY, which uses the Matrix Iteration method, is listed below. Example data input files for the COR-TEN freestanding tower and a double guyed Rohn 45 GSR tower are shown. The geometry and structural properties of the tower are read from this input file called "MITGDA."

The first value that must be read is the tower height in inches. Also, the tower must be divided into N segments, in order to approximate the mass and stiffness distributions. The value of N is the next value which is read. Then, the number of sets of guy wires is read. For the Rohn 45 GSR double guyed tower this number is two; for the unguyed COR-TEN tower this number is zero.

Next the stiffness, weight, and modeshape approximations must be read. The stiffness at each node point is read (lb-in.^{2}). Also, the weight distribution at each node point is read (lb/in.). The user should have plots or tables of the tower stiffness and weight distributions.

Recall that the matrix iteration method is based on the equation (see Section 2.5):

$$\frac{1}{W^2} \{\phi\} = [f] [m] \{\phi\}. \qquad E1)$$

To obtain the [f] matrix, the tower is divided into N equal segments and the stiffness properties are evaluated at the nodes (endpoints of each segment). The values of stiffness at the nodes are expressed as EI_i. The program then automatically approximates the tower as a series of constant stiffness steps $EI_{i,\ i-1}$ where:

$$EI_{i,\ i-1} = \frac{EI_i + EI_{i-1}}{2}$$

for i = 1,2,..., N. The elements of [f] are then calculated (see Ref. 7) by the program.

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To obtain the lumped mass matrix [m], one inputs the distributed weight at each node. The program then calculates a simple diagonal lumped mass matrix automatically.

The data values which are input are simply the tower stiffness and distributed weight at each node. Also, the amplitudes of the first two bending modeshape approximations must also be input at each node. As can be seen from the example input files, these values are simply chosen as ones. Even with these poor modeshape approximations, convergence to the first and second bending frequencies and modeshapes will proceed rapidly.

After the stiffness, weight, and modeshape data have been read, the value of the machine weight is read. This value is seen as 50 lb for the COR-TEN tower and 680 lb for the Rohn 45 GSR. The COR-TEN had no machine on top, just a top plate.

If an unguyed tower is being analyzed, the data input is now complete. For a double guyed tower, two sets of guy wire data must be input. As can be seen for the Rohn 45 GSR example, the first value that is read is the guy wire cross sectional area A'(in.^{2}). The next value is the guy wire elastic modulus E'(lb-in.^{2}). The length ℓ of the guy wires in each set is then input, also the angle (in radians) that the guy cables of a particular set make with the tower (assuming this angle to be the same for each guy wire in a set). Next, the node point at which the guy wires in a given set are attached must be specified. The user must be careful to divide the tower into the correct number of segments so that guy wires in a given set are attached at a node point. The first node point (node #1) is located at the tower base.

The data input has now been described. By matrix iteration methods (Section 2.5) the program will calculate and print both the first and second bending frequencies and modeshapes.

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```
00100 PROGRAM MITGUY(INPUT,OUTPUT,MITGDA,TAPE5=INPUT,
00110+TAPE6=OUTPUT,TAPE7=MITGDA)
00120 DIMENSION EI(50),MASS(50,50),E(50,50),W(50,2),C(50,50)
00130+,D(50,1),XG(3),GC(50,50)
00140C
00150C
00160C
00170C      THIS PROGRAM CALCULATES MODESHAPES AND
00180C      FREQUENCIES FOR FREE STANDING AND GUYED
00190C      TOWERS. THE GEOMETRY AND STRUCTURAL
00200C      PROPERTIES ARE INPUT FROM A FILE
00210C      CALLED # MITGDA #. THE THEORY IS TAKEN
00220C      FROM...
00230C
00240C
00250C           #WIND ENERGY CONVERSION#
00260C                DESIGN MANUAL
00270C                VOLUME 1
00280C              ASRL-TR-184-7
00290C       BY: R. MILLER
00300C           J. DUGUNDJI
00310C           M. MARTINEZ
00320C           J. GOHARD
00330C           S. CHUNG
00340C           J. WENDELL
00350C
00360C
00370C
00380C
00390 REAL MASS,NACELM,XG,LENGTH,MODULS
00400 INTEGER ELEMENT,GUYS
00410 A=0.
00420 B=0.
00430 D=0.
00440 READ(7,*) HIGHT,ELEMENT,GUYS
00450 DX=HIGHT/ELEMENT
00460 NODES=ELEMENT+1
00470 READ(7,*)(EI(N),MASS(N,N),W(N,1),W(N,2),N=1,NODES)
00480 READ(7,*) NACELM
00490C
00500C
00510C
00520C      INTEGRATE STIFFNESS PROPERTIES
00530C      AND CALCULATE THE FLEXIBILITY
00540C      COEFFICIENTS.
00550C
00560C
00570 DO 20 N=1,ELEMENT
00580 EI(N)=(EI(N+1)+EI(N))/2
00590 A=A+(DX**3)/EI(N)
```


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00600 B=B+(DX**3)*((N**2)-(N-1)**2)/(2*EI(N))
00610 D=D+(DX**3)*((N**3)-(N-1)**3)/(3*EI(N))
00620 DO 15 I=N,ELEMENT
00630 15 C(N,I)=(D-(N*B))-I*(B-(N*A))
00640 20 CONTINUE
00650 DO 25,N=1,ELEMENT
00660 DO 25,I=1,ELEMENT
00670 25 C(I,N)=C(N,I)
00680C
00690C
00700C
00710C     GUY WIRE STIFFNESS COEFFICIENTS ADDED TO
00720C         STIFFNESS MATRIX
00730C
00740 IF(GUYS.EQ.0.0) GO TO 29
00750 DO 28 N=1,GUYS
00760 READ(7,*)A,MODULS,LENGTH,ANGLE,NGUY
00770 NGUY=NGUY-1
00780 KG(N)=1.5*A*MODULS*((SIN(ANGLE))**2)/LENGTH
00790 DO 27 I=1,ELEMENT
00800 DO 27 J=1,ELEMENT
00810 27 GC(I,J)=C(I,J)-(KG(N)/(1+C(NGUY,NGUY)*KG(N)))*C(I,NGUY)*C(J,NGUY)
00820 DO 28 I=1,ELEMENT
00830 DO 28 J=1,ELEMENT
00840 28 C(I,J)=GC(I,J)
00850 29 CONTINUE
00860C
00870C
00880C
00890C
00900 30 FORMAT(6E9.2)
00910C
00920C
00930C   CALCULATE MASS MATRIX
00940C
00950C
00960 DO 50 M=1,ELEMENT
00970 N=M+1
00980 IF(N.EQ.NODES)40,45
00990 40 G=0.5
01000 GO TO 50
01010 45 G=1.0
01020 50 MASS(M,M)=DX*MASS(N,N)*G/384
01030 MASS(ELEMENT,ELEMENT)=MASS(ELEMENT,ELEMENT)+NACELM/384.
01040C
01050C
01060C
01070C        CALCULATE DYNAMIC MATRIX
01080C        THIS IS A LIBRARY SUBROUTINE
01090C        IN THE MATH SCIENCE LIBRARY.
01100C
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01110C
01120C
01130 CALL MTMLT(C,MASS,E,ELEMENT,ELEMENT,ELEMENT,ELEMENT)
01140C
01150C
01160C       ITERATE THE SOLUTION OF THE
01170C       DYNAMIC EQUATION
01180C
01190C
01200C
01210 AMP=0.
01220 55 CALL MTMLT(E,W,D,ELEMENT,ELEMENT,ELEMENT,1)
01230 DO 56 I=1,ELEMENT
01240 56 W(I,1)=D(I,1)
01250 AMPP=ABS(1/W(ELEMENT,1)-AMP)
01260 IF(AMPP.LT.0.01)70,60
01270 60 AMP=1/W(ELEMENT,1)
01280 DO 65 N=1,ELEMENT
01290 65 W(N,1)=W(N,1)/W(ELEMENT,1)
01300 GO TO 55
01310 70 FREQ=SQRT(1.0/W(ELEMENT,1))/(2.*3.14)
01320 DO 75 N=1,ELEMENT
01330 75 W(N,1)=W(N,1)/W(ELEMENT,1)
01340 PRINT 80,FREQ
01350 80 FORMAT("THE FUNDAMENTAL TOWER FREQUENCY IS",F10.2,"HZ")
01360 WRITE(6,85)
01370 85 FORMAT("THE MODE SHAPE IS...")
01380 WRITE(6,90)(W(N,1),N=1,ELEMENT)
01390 90 FORMAT(F8.3)
01400C
01410C
01420C          THIS SECTION SOLVES FOR THE
01430C          SECOND MODE BY IMPOSSING
01440C          ORTHOGONALITY ON THE FIRST MODE.
01450C
01460C
01470C
01480 AMP=0.
01490 100 CALL MODE2(MASS,W,ELEMENT)
01500 110 CALL MTMLT(E,W,D,ELEMENT,ELEMENT,ELEMENT,2)
01510 DO 120 I=1,ELEMENT
01520 120 W(I,2)=D(I,2)
01530 AMPP=ABS(1/W(ELEMENT,2)-AMP)
01540 IF(AMPP.LT.0.0001)140,130
01550 130 AMP=1/W(ELEMENT,2)
01560 DO 135 N=1,ELEMENT
01570 135 W(N,2)=W(N,2)/W(ELEMENT,2)
01580 GO TO 100
01590 140 FREQ2=SQRT(1.0/W(ELEMENT,2))/(2.*3.14)
01600 DO 150 N=1,ELEMENT
01610 150 W(N,2)=W(N,2)/W(ELEMENT,2)
```


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01620 PRINT 160,FREQ2
01630 160 FORMAT("THE SECOND TOWER BENDING FREQUENCY IS",F10.2,"HZ")
01640 WRITE(6,85)
01650 WRITE(6,90)(W(N,2),N=1,ELEMENT)
01660 END
01670 SUBROUTINE MTMLT(A,B,C,IA,JA,IB,JB)
01680 DIMENSION A(50,50),B(50,50),C(50,50)
01690 DO 20 I=1,IA
01700 DO 20 J=1,JB
01710 C(I,J)=0.0
01720 DO 20 K=1,JA
01730 20 C(I,J)=C(I,J)+A(I,K)*B(K,J)
01740 RETURN
01750 END
01760C
01770C
01780C      ORTHOGONALITY CORRECTION ROUTINE.
01790C
01800C
01810 SUBROUTINE MODE2(MASS,W,IA)
01820 DIMENSION MASS(50,50),W(50,2)
01821 REAL MASS
01822 SUM=0.0
01830 IB=IA-2
01835 IC=IA-1
01840 DO 10 I=1,IB
01850 10 SUM=SUM+MASS(I,I)*W(I,1)*W(I,2)
01860 W(IC,2)=-(SUM+(MASS(IA,IA)*W(IA,1)*W(IA,2)))
01870+ /(MASS(IC,IC)*W(IC,1))
01880 RETURN
01890 END
 EOI ENCOUNTERED.
/
```

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 81/10/12. 14.38.33.
PROGRAM   CORTEN

660.0,20,0
1.140E11,8.72,1.,1.
1.029E11,8.45,1.,1.
9.160E10,8.16,1.,1.
8.150E10,7.84,1.,1.
7.140E10,7.51,1.,1.
6.290E10,7.19,1.,1.
5.440E10,6.87,1.,1.
4.750E10,6.55,1.,1.
4.050E10,6.20,1.,1.
3.480E10,5.88,1.,1.
2.910E10,5.55,1.,1.
2.460E10,5.23,1.,1.
2.020E10,4.91,1.,1.
1.670E10,4.59,1.,1.
1.320E10,4.27,1.,1.
1.060E10,3.94,1.,1.
8.035E9,3.62,1.,1.
6.250E9,3.30,1.,1.
4.450E9,2.98,1.,1.
2.650E9,2.66,1.,1.
2.137E9,2.34,1.,1.
50.

 READY.
```


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```
 81/10/12. 14.33.58.
PROGRAM  450SR

480.,40,2
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.417,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,2.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
5.800E9,1.667,1.,1.
680.
.1169,18000000.0,470.4,.912,.35
.0597,20000000.0,526.0,.785,.32
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